Regional Mathematical Olympiad - 2026

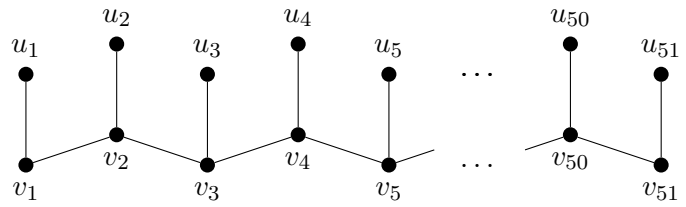
Time: 3 hours **November 15, 2026**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **All questions carry equal marks. Maximum marks: 102.**
- **No marks will be awarded for stating an answer without justification.**
- **Answer all the questions.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Let ABC be a triangle with $AB = AC$. Let D be the midpoint of BC and P be a point on the segment AB satisfying $\angle CPA < 90^\circ$. Suppose Q ($\neq P$) is a point on the circumcircle of triangle BPC such that $\angle AQC = 90^\circ$. Prove that $\angle DQP = 90^\circ$.

2. Consider the following diagram with 102 vertices:



   Show that one can assign the numbers $1, 2, \ldots, 102$ to the 102 vertices $u_1, u_2 \ldots, u_{51}$, $v_1, v_2, \ldots, v_{51}$ such that no two vertices have the same number, and the absolute differences of the numbers on adjacent vertices is a rearrangement of $1, 2, \ldots, 101$. (Two points are adjacent if they have a line segment joining them in the above diagram.)

3. Prove that for any two real numbers x and y, we have

$$\left(x^6 - x^2 + 3\right)\left(y^6 - y^2 + 3\right) > 2\left(x^2 + y^2\right)^2.$$

4. Let ABC be an acute triangle, and let M and N be the midpoints of AB and AC, respectively. Let T be the point of intersection of ray NM with the circumcircle of ABC. Suppose that the circumcircle of ATM is tangent to AC at A. Let S be the foot of perpendicular from C on AB. Determine the value of AS/SB.

5. Let a be an integer greater than 1. Determine all positive integers b which have the property that $a^b + 1$ divides a number of the form $a^n + b$ for some positive integer n.

6. Let n be a fixed positive integer and S be a fixed positive real number. Find the minimum constant c (in terms of S, n) such that:

   Given any positive real numbers $a_1, a_2, \ldots, a_n$ with $a_1 + a_2 + \cdots + a_n = S$, there is a rearrangement $b_1, b_2, \ldots, b_n$ of $a_1, a_2, \ldots, a_n$ such that

$$(a_1 + b_1) + (a_2^2 + b_2^2) + \cdots + (a_n^n + b_n^n) \leq c.$$

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