Regional Mathematical Olympiad - 2026

Time: 3 hours **November 15, 2026**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **All questions carry equal marks. Maximum marks: 102.**
- **No marks will be awarded for stating an answer without justification.**
- **Answer all the questions.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. The following three points related to an acute scalene triangle ABC are given: vertex A, orthocenter H, the midpoint M of side BC. Describe with justification how to construct triangle ABC using straightedge and compasses.

2. An integer $n \geq 2$ is called *good* if for any n non-zero integers $a_1, a_2, \ldots, a_n$ (not necessarily distinct) satisfying

$$\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n} = 0,$$

there exist $i, j \in \{1, 2, \ldots, n\}$, $i \neq j$ such that $\gcd(a_i, a_j) > 1$. Find all good integers.

3. Let ABC be an isosceles triangle with $AB = AC$ and $\angle BAC = 80^\circ$. Let P be an interior point of triangle ABC such that $\angle PAB = 10^\circ$ and $\angle PCB = 20^\circ$. Prove that $PA = PB$.

4. Find all positive integers x, y, z such that

$$2^x + 5^y = z^2.$$

5. Suppose a, b, c are distinct non-zero real numbers such that $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 0$ and $\dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2} = 3$. Prove that

$$\left(\frac{ab}{a-b}\right)^2 + \left(\frac{bc}{b-c}\right)^2 + 16\left(\frac{ca}{c-a}\right)^2 > 4.$$

6. Akash and Bhumi play a game. They start with a 45×45 square board with 2025 uncoloured unit cells. Each player on their turn selects a row or column and then colours all uncoloured cells in that chosen row or column with their colour. Akash colours with red and Bhumi with blue. If Akash goes first, find the maximum number of cells Bhumi can colour with blue assuming both players want to maximize cells of their colour and play optimally.

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