

# Indian National Astronomy Olympiad (INAO) – 2026

## Question Paper

Roll Number:  -  -

Date: 31 January 2026

Duration: **Three Hours**

Maximum Marks: 100

### *Please Note:*

- Before starting, please ensure that you have received a copy of the question paper containing total 8 pages (4 sheets).
- Please write your roll number in the space provided above.
- There are total 6 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answersheet must be returned to the invigilator.** You can take this question paper back with you.

### Useful Constants

Mass of Sun

$$M_{\odot} = 1.988 \times 10^{30} \text{ kg}$$

Radius of Sun

$$R_{\odot} = 6.957 \times 10^8 \text{ m}$$

Luminosity of Sun

$$L_{\odot} = 3.828 \times 10^{26} \text{ W}$$

Solar constant (above atmosphere of Earth)

$$S_{\odot} = 1361 \text{ W m}^{-2}$$

Mass of Earth

$$M_{\oplus} = 5.972 \times 10^{24} \text{ kg}$$

Radius of Earth

$$R_{\oplus} = 6.378 \times 10^6 \text{ m}$$

Speed of light in vacuum

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$

Stefan-Boltzmann constant

$$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Wien's displacement constant

$$b = 2.898 \times 10^{-3} \text{ m K}$$

Universal gravitational constant

$$G = 6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

1 Astronomical Unit

$$1 \text{ au} = 1.496 \times 10^{11} \text{ m}$$

1. One of Galileo's discoveries was the satellites of Jupiter and in his notebook he drew sketches showing the position of the satellites every night. In this picture of his original hand-written notes, the big circle denotes Jupiter and small stars represent the positions of four satellites. The writing on the left is the date.

Observations Jupiter			
20. Jan.	Mar 11. 1610	○	***
30. Jan.		***○	*
2. Feb.		○***	*
3. Feb.		○***	
3. Feb. 5.		*○	*
4. Feb.		*○	**
6. Feb.		**○	*
8. Feb. 13.		***○	
10. Feb.		* * * ○	*
11.		* * ○ *	
12. Feb.		* ○ *	
17. Feb.		* * ○ *	
14. Feb.		* * ○ *	

(a) (10 marks) The following table gives recent measurements of the position of one of the satellites, Europa, with respect to Jupiter at various times. Here,  $x$  represents the magnitude of distance between Jupiter and Europa, as measured in a similar image / sketch. Obtain the time period ( $T$ ) of Europa through a suitable linear plot. (You may assume that the maximum magnitude of  $x$  is 3 cm).

$T$ (in hour)	$x$ (in cm)	$T$ (in hour)	$x$ (in cm)
0	not seen	9	2.02
1	0.49	10	2.18
2	0.65	11	2.32
3	0.84	12	2.45
4	1.06	13	2.56
5	1.26	14	2.67
6	1.45	15	2.78
7	1.65	16	2.84
8	1.82	17	2.91

(b) (2 marks) Draw a rough sketch of the same plot as in part (a), for a full period of Europa.

2. (a) (6 marks) An observer is located in the city of Nashik (latitude  $\approx 20^\circ\text{N}$  and longitude  $\approx 73^\circ\text{E}$ ). She observes the rising of the Sun on different days of the year.

The figure in the answersheet depicts the eastern horizon (approximated as a straight line) for the city of Nashik with East cardinal point marked as E. The azimuth range is given to be  $60^\circ$  to  $120^\circ$  with markings at every  $10^\circ$ . In the table below, you are given certain dates alongside alphabets. Mark the approximate rising points of the Sun as seen by the observer for these dates on the image of the horizon given in the answersheet and label them with the corresponding alphabets. Precise calculations are not expected.

Note: For definition of Azimuth refer Appendix.

Letter	Date
A	01 Jan 2025
B	01 Apr 2025
C	15 May 2025

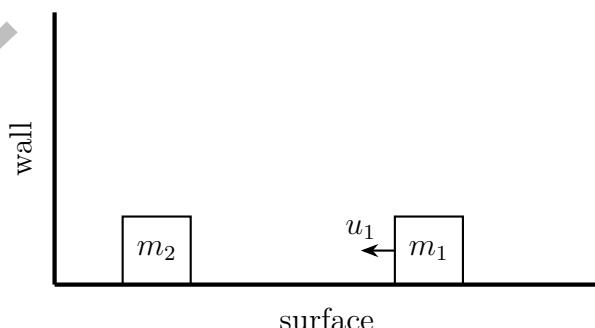
Letter	Date
D	01 Jun 2025
E	01 Jul 2025
F	01 Oct 2025

(b) (4 marks) Now consider an observer who is located in a city which lies on the equator of Earth. The figure given in the answersheet is of the horizon (approximated as a straight line) as seen from the equator in the azimuth range  $180^\circ$  to  $360^\circ$ . The cardinal west point is marked with letter W. The azimuths are marked at a separation of  $30^\circ$ .

Mark the approximate setting points of the following stars (by writing their corresponding Sr. No.), if applicable, on the figure of horizon given in your answersheet.

Sr. No.	Star Common Name	Bayer Name
1	Pollux	$\alpha$ Gem
2	Polaris	$\alpha$ UMi
3	Canopus	$\alpha$ Car
4	Vega	$\alpha$ Lyr
5	Revati	$\zeta$ Psc
6	Rigel	$\beta$ Ori
7	Dubhe	$\alpha$ UMa
8	Kaus Borealis	$\lambda$ Sgr

3. Two blocks,  $m_1$  and  $m_2$ , are placed on a frictionless horizontal surface next to a fixed rigid wall. Block  $m_2$  is at rest close to the wall and block  $m_1$  moves towards it with velocity  $u_1 = -1 \text{ m s}^{-1}$ . All collisions (between the blocks and with the wall) are perfectly elastic.



(a) (5 marks) In the first case, we consider two identical blocks, each of mass 1 kg. Calculate total number of collisions (block-block or block-wall),  $n_1$ , that will occur in this system.

(b) (2 marks) Now, we will attempt to describe this motion in velocity space by drawing an appropriate figure but for a more general problem.

In the coordinate grid given in the answersheet, we redefine the two axes as,  $\alpha = v_1\sqrt{m_1}$  and  $\beta = v_2\sqrt{m_2}$ .

Plot the values of  $\alpha$  and  $\beta$  that depict the velocities between successive collisions during each phase and connect them with straight line arrows depicting the transition at the instance of each collision.

(c) (3 marks) On this phase diagram, we can plot several constant energy contours. Draw the constant energy contour, on the same grid given in part (b), that passes through the phase points that you have plotted.

(d) (6 marks) Consider a general case with arbitrary values of  $m_1$  and  $m_2$ . In the phase diagram for this general case, let points  $a$ ,  $b$ ,  $c$  be the first three phase points. Let  $\angle abc = \theta$ . Find expression for  $\theta$  in terms of  $m_1$  and  $m_2$ .

(e) (3 marks) Using this information from part (d) draw a complete phase diagram for  $m_1 = 4m_2$ .

(f) (3 marks) You may have realised, in a general case, i.e. for an arbitrary ratio  $m_1/m_2$ , you will be able to define a region of this curve in which the last phase point must lie. If  $p$  and  $q$  are the end points of that region on the curve and  $o$  is the origin, find equations of lines  $op$  and  $oq$ .

Draw these lines for  $m_1/m_2 = 4$  case in the diagram of part (e).

*Hint:* In one of the two equations, you will be using the ratio  $\frac{m_2}{m_1}$ .

(g) (5 marks) Write an inequality showing bounds on  $n$ , where  $n$  is the total number of collisions, in terms of  $m_1$  and  $m_2$ .

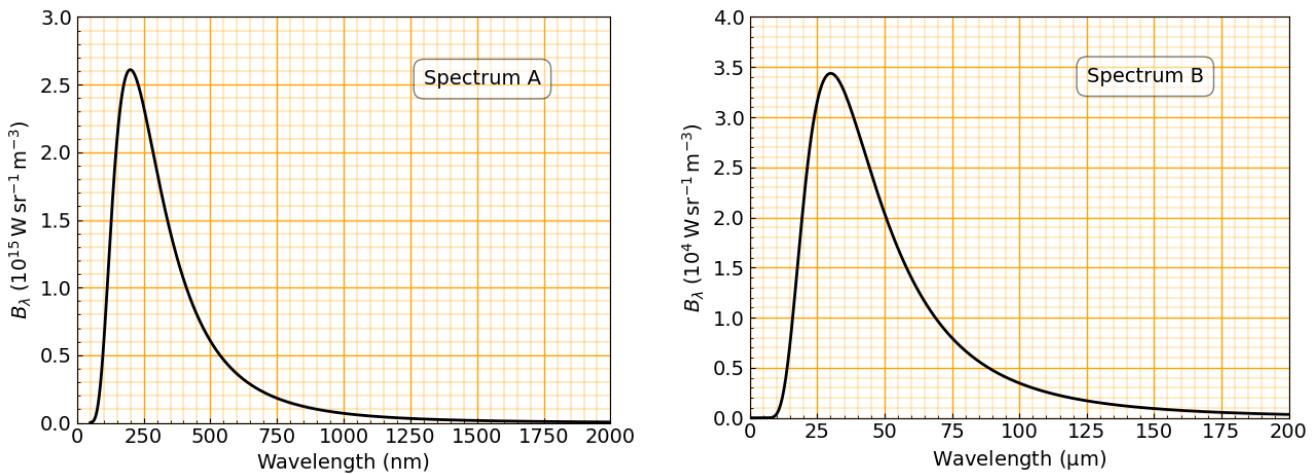
*Hint:* Find an inequality between  $\theta$  and  $n$ .

(h) (3 marks) Lastly, use the information above to estimate the total number of collisions,  $n_{\text{total}}$ , for the case  $m_1 = 10^{10}m_2$ .

#### 4. Blackbody Radiation

(a) (4 marks) Consider a star of radius  $R_s = 3R_\odot$  and a gas cloud of radius  $R_g$  at a distance  $d$  from the star. Assume that the gas cloud doesn't contain any source of radiation and there are no other stars in the vicinity of this system. Both the star and the gas cloud can be assumed to be blackbodies.

You are given two spectra (A and B) below. Determine which spectra corresponds to the star and which corresponds to the gas cloud. Justify your answer. Also, calculate the effective temperature for the star,  $T_s$ , and for the gas cloud,  $T_g$ .



(b) (4 marks) Calculate the distance,  $d$ , of the gas cloud from the star.

### 5. (10 marks) Observing the ISS

The International Space Station (ISS) is sometimes visible in the sky, during the morning and evening twilight, as a bright moving object. One day, Kundan sees the ISS rising at the horizon, then passing near the zenith, and then setting in the opposite direction.

Consider the ISS to be  $w$  metre across and orbiting the Earth in a circular orbit at a height  $x$  metre above the Earth's surface.

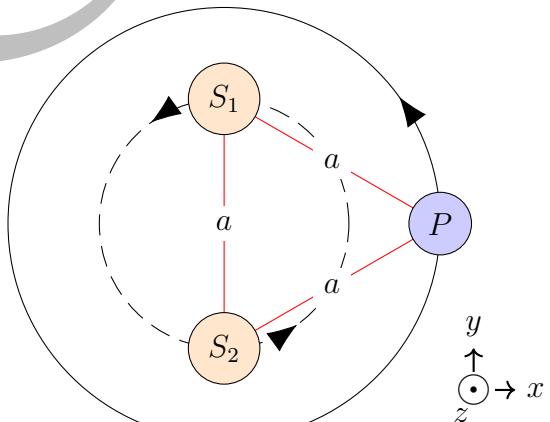
Derive an expression for its apparent angular size,  $\phi$ , measured by Kundan as a function of its altitude from Kundan's location.

### 6. Planet's Orbit around a Binary Star System:

Dhananjay, an exoplanet researcher, wanted to explore the prospects of life on an Earth like planet in a binary star system. More specifically he wanted to look into the following configuration: two identical stars  $S_1, S_2$  and a planet  $P$  have circular orbits about the center of mass of the system and at every point in time, the three bodies form a equilateral triangle with side of length  $a = 2 \text{ au}$  as shown in the figure below. (Note that at the instance shown in this figure, the radius vector from the centre of mass of the system to  $P$  is exactly along the  $x$ -axis).

The period  $t_P$  of such a system is known to be:

$$t_P = \sqrt{\frac{4\pi^2 a^3}{G(m_1 + m_2 + m_3)}}$$

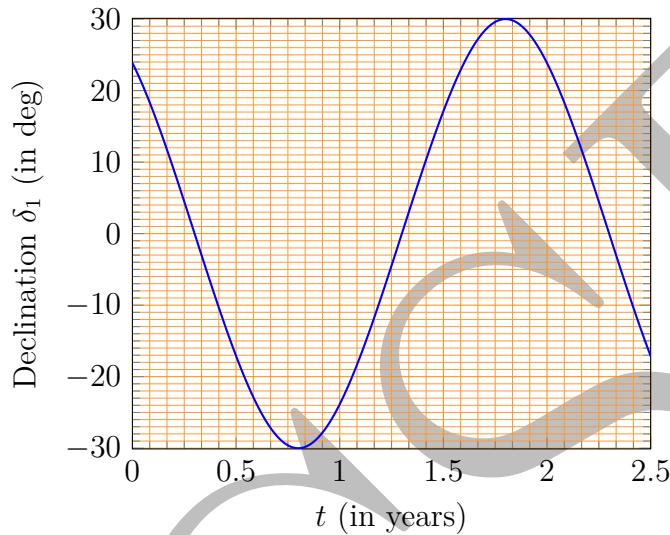


You can take the mass and radius of the planet  $P$  to be exactly that of Earth and the two stars are Sun like. The sidereal period of rotation of  $P$  is 1 d and the angle made by the  $P$ 's axis of rotation with the  $z$ -axis in the diagram above, called the axial tilt, is  $\epsilon = 30^\circ$ .

Help Dhananjay answer the following questions about the planet:

(a) (3 marks) Taking the albedo of the planet  $P$  to be  $A = 0.15$ , estimate its average surface temperature ( $T_P$ ).

Dhananjay calculated the variation of declination,  $\delta_1$ , of star 1 with time as seen from the planet. This variation is plotted in the figure below.



Note that declination is defined in the usual sense - the angle made by the object with the celestial equator of the planet.

(b) (4 marks) Plot the declination,  $\delta_2$ , of star  $S_2$  vs time in the plot provided in answersheet.  
 (c) (6 marks) The configuration of the two stars and planet system shown in the figure given in the preamble of the question is the orientation of the system corresponding to time  $t = 0$  in the declination vs time plot.

In the figure given in the answersheet, mark the direction of the equinoxes, as seen from the planet, with respect to the  $x$ -axis on the orbit of planet. Mark the equinox after which the days get longer, in the northern hemisphere of the planet, with a  $\times$  and write VE beside it and the other one with  $\Delta$  and write AE beside it.

Note that the equinox is the day corresponding to the median day length.

(d) (10 marks) We define the effective flux at a point on the planet as the energy per unit area per unit time falling on a tangential plane at that point. Plot the effective flux on the planet's North Pole from  $t = 0$  yr to  $t = 2.5$  yr.  
 (e) (4 marks) What will be the typical day length at the equator,  $t_{\text{day, eq}}$ , and the pole,  $t_{\text{day, pole}}$ ? Explain your answer.  
 (f) (3 marks) What is/are the number of zero shadow days,  $N_{\text{ZSD}}$  for an observer on the planet in one period of the system for the following two cases -

Case 1: Latitude  $> 30^\circ$

Case 2: Latitude  $< 30^\circ$

Explain your answer in brief.