Regional Mathematical Olympiad - 2025

Time: 3 hours **November 16, 2025**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **All questions carry equal marks. Maximum marks: 102.**
- **No marks will be awarded for stating an answer without justification.**
- **Answer all the questions.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. (a) Let $n \geq 3$ be an integer. Find a configuration of n lines in the plane which has exactly
   (i) $n-1$ distinct points of intersection;
   (ii) n distinct points of intersection;

   (b) Give configurations of n lines that have exactly $n+1$ distinct points of intersection for (i) $n = 8$ and (ii) $n = 9$.

2. Let a, b, c be distinct nonzero real numbers satisfying
$$a + \frac{2}{b} = b + \frac{2}{c} = c + \frac{2}{a}.$$
Determine the value of $|a^2b + b^2c + c^2a|$.

3. Let Ω and Γ be circles centred at O_1, O_2 respectively. Suppose that they intersect in distinct points A, B. Suppose O_1 is outside Γ and O_2 is outside Ω. Let ℓ be a line not passing through A and B that intersects Ω at P, R and Γ at Q, S so that P, Q, R, S lie on the line in this order. Furthermore, the points O_1, B lie on one side of ℓ and the points O_2, A lie on the other side of ℓ. Given that the points A, P, Q, O_1 are concyclic and B, R, S, O_2 are concyclic as well, prove that $AQ = BR$.

4. Prove that there do not exist positive rational numbers x and y such that
$$x + y + \frac{1}{x} + \frac{1}{y} = 2025.$$

5. Let ABC be an acute-angled triangle with $AB < AC$, orthocentre H and circumcircle Ω. Let M be the midpoint of minor arc BC of Ω. Suppose that MH is equal to the radius of Ω. Prove that $\angle BAC = 60°$.

6. Let $p(x)$ be a nonconstant polynomial with integer coefficients, and let $n \geq 2$ be an integer such that no term of the sequence
$$p(0),\ p(p(0)),\ p(p(p(0))), \ldots$$
is divisible by n. Show that there exist integers a, b such that $0 \leq a < b \leq n-1$ and n divides $p(b) - p(a)$.

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