Indian National Physics Olympiad (INPhO)-2025

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Question Paper

Date: 02 February 2025
Time: **09:00-12:00 (3 hours)** Maximum Marks: **75**

Instructions **Roll Number:** ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐

1. This booklet consists of 9 pages and total of 5 questions. Write roll number at the top wherever asked.

2. Booklet to write the answers is provided separately. Instructions to write the answers are on the Answer Booklet.

3. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.

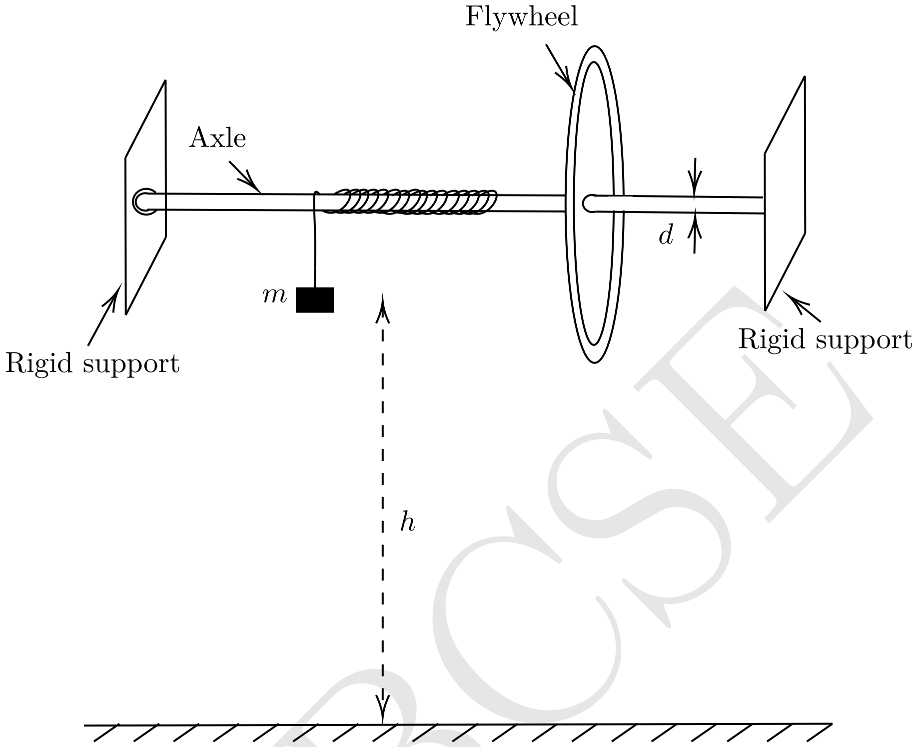
4. **Please submit the Answer Sheet at the end of the examination.** You may retain the Question Paper.

Table of Constants

Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Magnitude of electron charge	e	1.60×10^{-19} C
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Boltzmann constant	k_B	1.3806×10^{-23} J·K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H·m^{-1}

Question Number	**1**	**2**	**3**	**4**	**5**	**Total**
Maximum Marks	8	14	16	22	15	**75**

1. **The Flywheel Chronicles**
   In the following experiment we are interested in determining the moment of inertia of a flywheel. The free ends of the axle in a flywheel assembly are placed inside grooves at both ends, to rigid supports provided on the wall (see diagram below). The diameter of the axle is $d = 2.72$ cm. The total work done by the axle in overcoming the friction in the two grooves per rotation is W. A massless string, attached to a point mass m is wound tightly in n turns (without overlap) on the axle. The string unwinds from the axle without slipping as the mass descends from an initial height h. The length of the string is adjusted such that when the mass just touches the floor, the string detaches from the axle.



   From the instant when the mass touches the floor (taken as $t = 0$), the flywheel continues to rotate, adding another N number of rotations before coming to rest in time $t = T$. The figure is not to scale.

   (a) **[5 marks]** Derive an expression for the moment of inertia I of the flywheel in terms of m, N, T, and other known parameters.

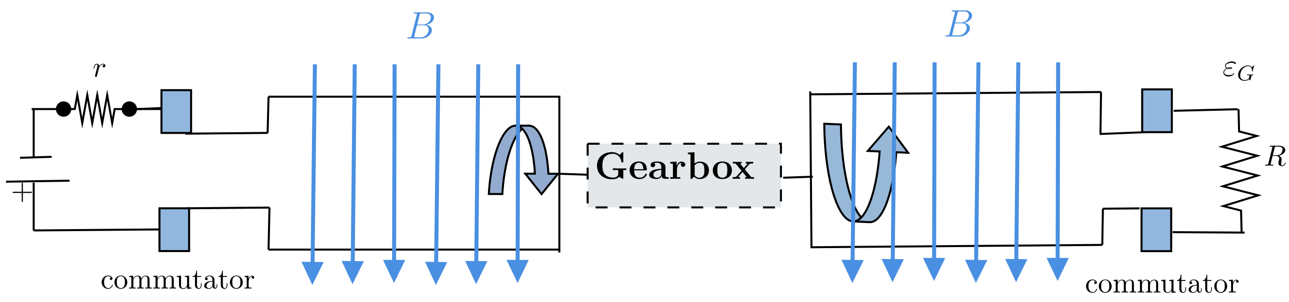
   (b) **[3 marks]** The following data has been obtained in this experiment.

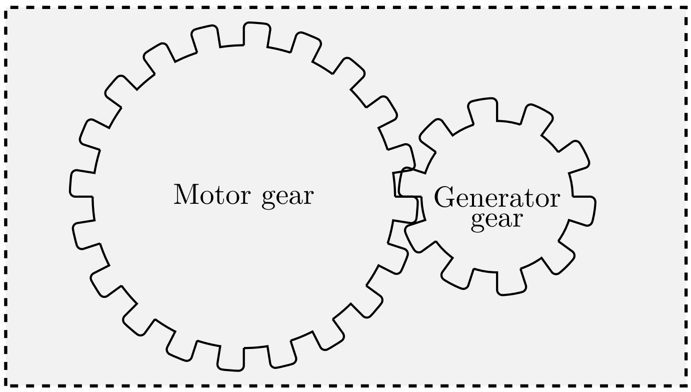
m (gm)	N	T (sec)
150.0	145.25	190.0
200.0	200.00	225.5
250.0	238.50	235.5

   Calculate the value of I for $n = 16$, and $h = 139$ cm.

2. **Gearminator: Rise of the Machines**

   We consider a "thought experiment" involving a DC motor and a DC generator coupled mechanically through a gearbox, operating under idealized conditions, to explore the power output and efficiency of the system (see schematic figure below). The schematic gearbox assembly is also shown in the figure.





Gearbox

Both the motor and the generator have N loops of area A and rotate in a uniform magnetic field of strength B. As usual, both the motor and the generator use commutators (indicated by the blue blocks) to reverse the direction of current in each arm every half cycle, to ensure unidirectional output. The generator is connected to an external resistance R, and the motor is driven by a constant voltage V_M with an internal resistance r. The gearbox is idealized, with no energy loss due to friction or otherwise, and no slipping between the teeth of the gears. For a pair of meshing gears, as shown above, the angular speed ratio, also known as the gear ratio X, is defined as:

$$X = \frac{\omega_M}{\omega_G},$$

where ω_M and ω_G are the angular velocities of the motor and the generator, respectively. Let $\langle P_G \rangle$, and $\langle P_M \rangle$ be the time-averaged generator output power and the time-averaged motor input power, respectively, over one complete cycle.

(a) **[6 marks]** Derive the expression for ω_G in terms of X, R, r, and the given parameters. For fixed values of r and R, determine the expression of X for which ω_G is maximum.

(b) **[3 marks]** Derive the expression for the generator output power $\langle P_G \rangle$ in terms of X, R, r, and the given parameters. For fixed values of r and R, determine the expression of X for which $\langle P_G \rangle$ is maximum.

(c) **[5 marks]** For fixed values of r and R, determine the condition on X for which the efficiency η is maximum, where

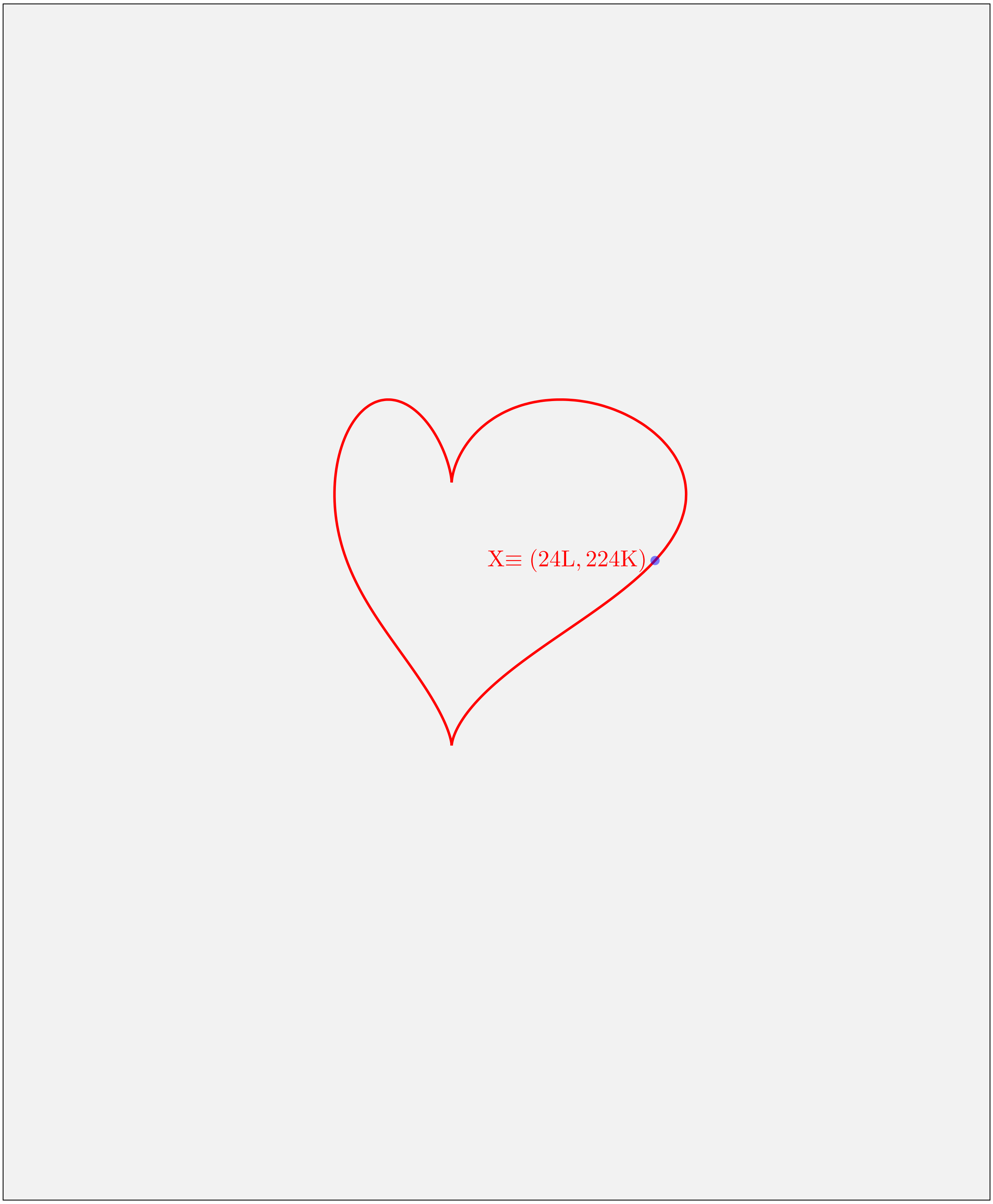
$$\eta = \frac{\langle P_G \rangle}{\langle P_M \rangle}.$$

Calculate this maximum value of η.

3. **Love is in the air**

A thermodynamic cycle is performed for one mole of an ideal monoatomic gas. The representation of this cycle is in the shape of a "heart" in the volume (V) – temperature (T) graph (shown as the shaded area below). However, neither of the axes are provided in the graph. The graph is drawn to scale with 1 cm along the V-axis representing 4 litre, and 1 cm along the T-axis representing 80 K.

It is given that the pressure at the point X is minimum for the whole cycle, with the temperature and volume at this point being $T_X = 224\,\mathrm{K}$ and $V_X = 24\,\mathrm{litre}$.



(a) **[13 marks]** Draw both the V and T axes to scale in the same diagram given in the Summary Answersheet. Indicate the origin by "O". Justify your answer in the detailed answersheet. You are given one extra answer box in the answersheet, in case of any mistake in the first.

(b) **[3 marks]** For the axes and origin you have drawn, indicate the point(s) on the graph where the pressure is/are maximum in the cycle by $\otimes$ and label it as P_{max} on the curve. Determine the value of the maximum pressure.

4. **The Magnetic Black Box (MBB)**
A magnetometer is a Hall-effect-based sensor that measures the magnetic field at its location. In the figure below, a magnetometer is located somewhere inside a closed "magnetic black box" (which we shall henceforth refer to as MBB) of negligible thickness. Fig. (1) gives a top view, where the red rectangle depicts the MBB. The plane of the rectangle is taken as the x–y plane of coordinates, with the origin O taken at the top right corner. The unknown location of the magnetometer is denoted by the coordinates (x_0, y_0). For example, it could be located at the

position marked by ⦿ inside the MBB. Note that the actual location of the magnetometer inside the MBB may be different from that in the figure; this is true for all subsequent figures in this problem as well.

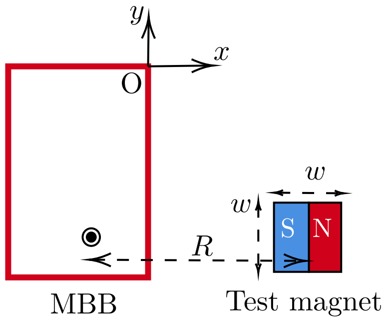


Figure 1: Schematic representation of the magnetic black box (MBB) and a test magnet.

The components B_x, B_y and B_z of the magnetic field measured by the magnetometer depend on the strength and the orientation of the magnetic dipole moment, of a magnet positioned nearby and the distance R between the center of the magnet and the magnetometer. The effect of the Earth's magnetic field is neglected throughout this problem.

Vanya is performing an experiment using the MBB. She has to first locate the exact position of the magnetometer inside the MBB. She has a cubical test magnet of side length $w = 10\,\text{mm}$ (see Fig. (1)) and unknown dipole strength $\vec{P}$.

She places the MBB on a wooden table. Then she records the magnetic field values displayed by the magnetometer as the test magnet is moved either parallel to the y-axis while keeping x fixed (**vertical scan**) or parallel to the x-axis while keeping y fixed (**horizontal scan**) as shown in Fig. (2). The magnitudes of the distances, r_x and r_y, measured from the center of the magnet, are also shown in the figure.

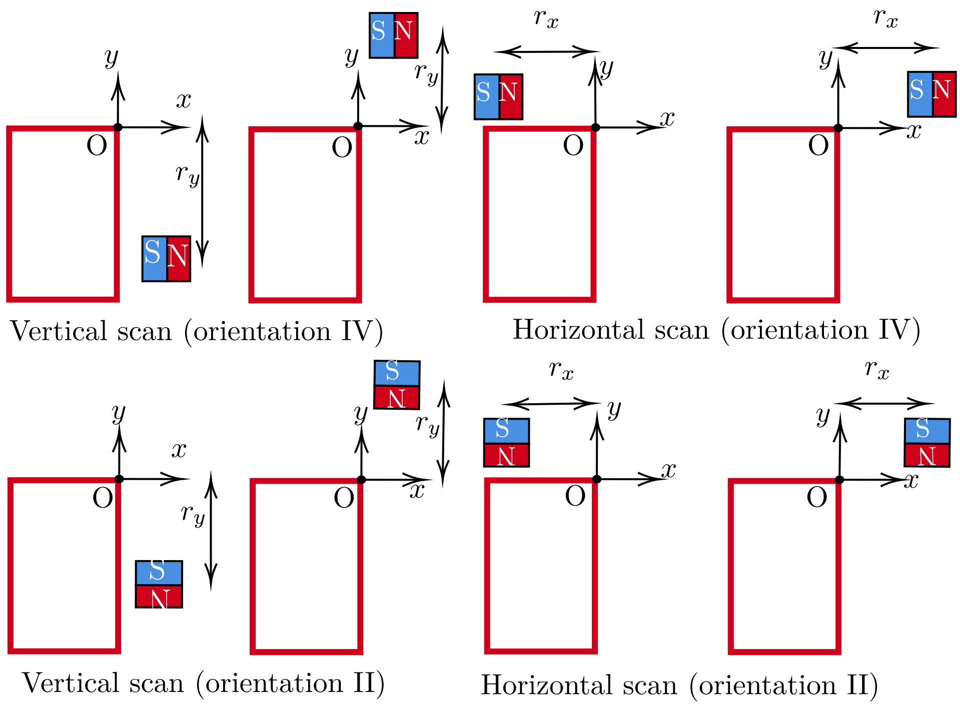


Figure 2: Some of the configurations of the vertical and horizontal scans as seen from the top. See Fig. (3) for the explanation of orientations.

For each scan, Vanya also tries different orientations of the magnet by aligning the dipole moment vector $\vec{P}$ either parallel or anti-parallel to the y-axis or x-axis. The different orientations (I to IV) are shown in Fig. (3). During the experiment, assume that the magnetometer location and the magnet's center are at the same height (i.e., their z-coordinates are always the same).

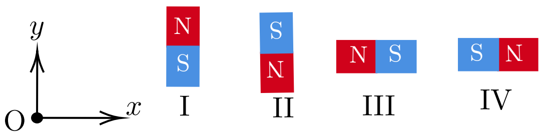
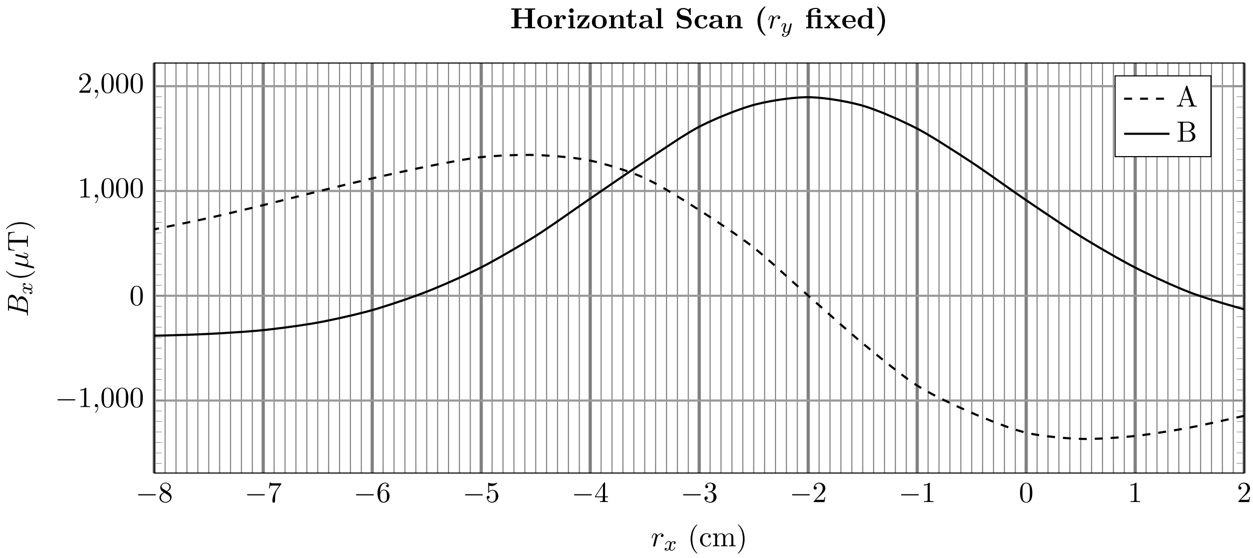


Figure 3: Different orientations of the test magnet

The graphs in Fig. (4) display the variation of the magnetic field B_x for four of the vertical and horizontal scans (denoted by A, B, C, D) with certain combinations of the orientations.



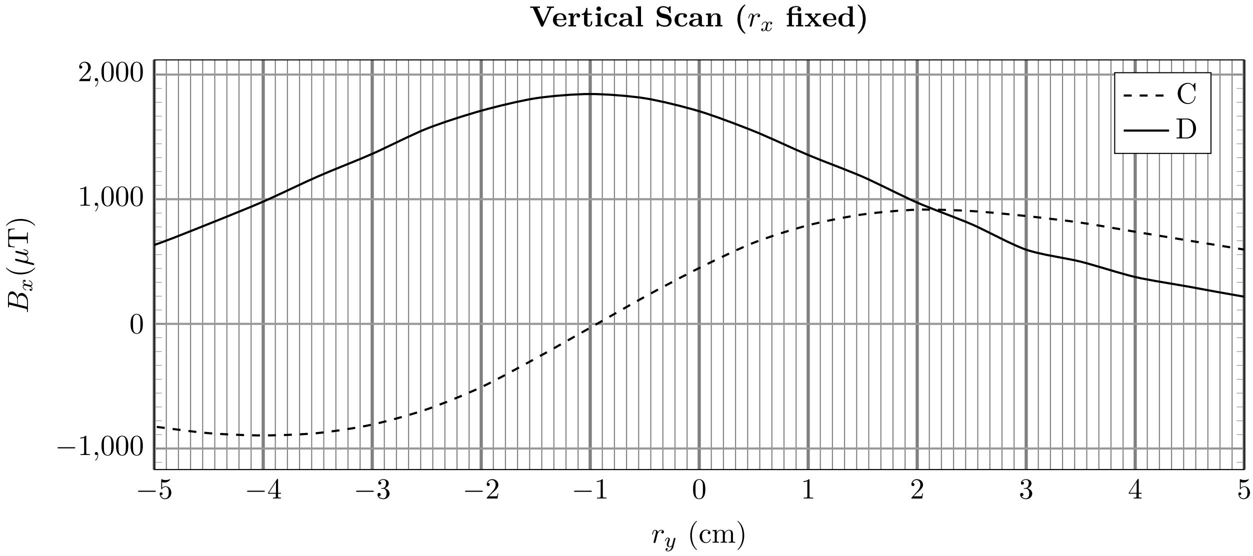


Figure 4

(a) **[7 marks]** Based on the above plots, identify which orientations (I-IV) these curves belong to. To indicate your answer, fill in the table in the answersheet. Determine the coordinates (x_0, y_0) of the magnetometer's position. You must justify your answers.

(b) Vanya is given two cuboidal magnetic sets M1 and M2, each constructed using two identical cubic magnets (of side length $w = 10\,\text{mm}$). In set M1, two magnets, each of dipole moment P', are joined in an attractive configuration. In set M2, the magnets are joined in a repulsive configuration using a strong adhesive. Thus, each magnetic set has a length of $2w$ (as shown in Fig. (5)).

Vanya aligns the central axis (XX' in Fig. (5)) of one magnetic set M1 or M2 such that the central axis passes through the magnetometer and is parallel to the x-axis. A representation of the setup is shown in Fig. 6. By keeping the y-coordinate fixed at y_0, Vanya moves the magnetic set parallel to the x-axis. The distance from the magnetometer to the midpoint of

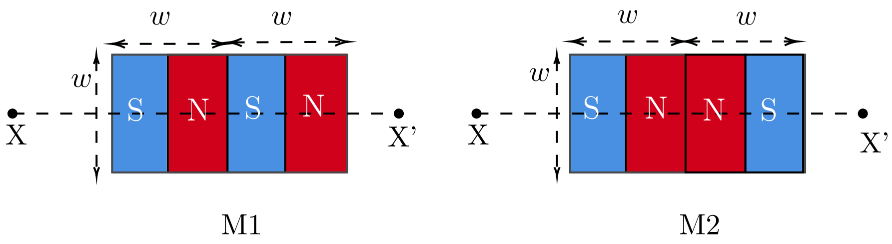


Figure 5: Magnetic sets M1 and M2. Here $w = 10$mm.

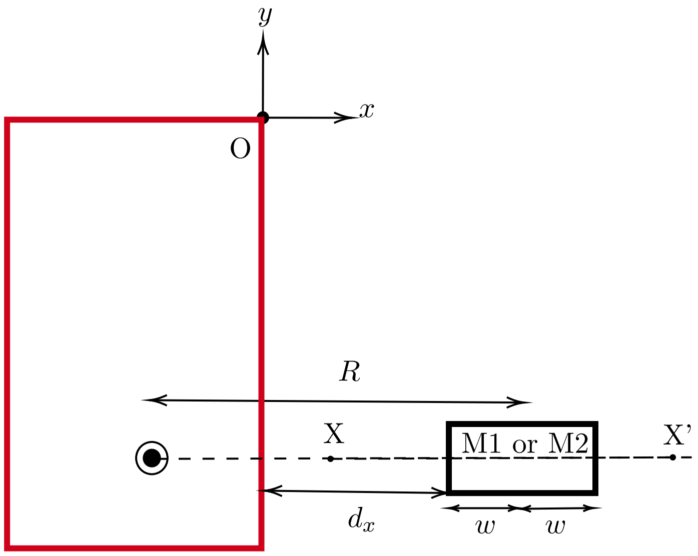


Figure 6: Setup for measuring the dipole moment.

the magnetic set is R. For each position, she measures the distance d_x (the distance from the face of the MBB to the nearest edge of the magnetic set) and the corresponding magnetic field component, B_x.

i. **[5 marks]** For the case of $R \gg w$, obtain expressions for the net magnetic field B at the magnetometer due to M1 and M2 in terms of R, w, P', and other constants. You may assume that each individual magnet can be modelled as a pair of magnetic monopoles separated by a distance w.

ii. **[10 marks]** One set of Vanya's data is presented in the table below.

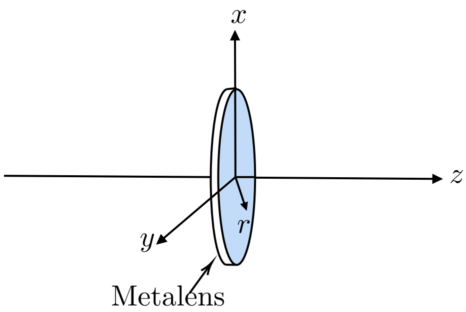
d_x (cm)	B_x (μT)	d_x (cm)	B_x (μT)
2.1	-1359	3.1	-646
2.3	-1168	3.3	-563
2.5	-1001	3.5	-493
2.7	-855	3.7	-447
2.9	-743	3.9	-398

Plot a suitable linear graph to analyze the data, and from the graph, identify whether the data belongs to M1 or M2. Justify your answer. From the same linear plot (or a different one), calculate P' of the individual magnets used in constructing the magnetic set.

5. **Metalens**

A metasurface is a two-dimensional, ultra-thin optical structure consisting of an array of nanospaced optical nano-elements (also known as meta-atoms) on a flat surface (typically an ultra-thin glass plate). The primary function of the nano-elements is to locally introduce a phase shift $\phi(\vec{\mathbf{r}})$ to the wave, incident at position $\vec{\mathbf{r}}$. This function, $\phi(\vec{\mathbf{r}})$, is called the phase profile of the metasurface.

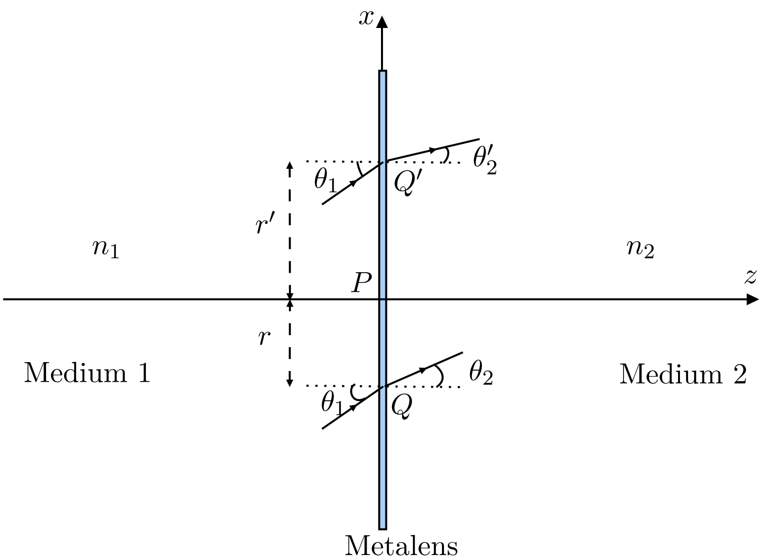
Metalens has a circular metasurface with a circularly symmetric phase profile function, $\phi(r)$, which depends on the distance r of the point from the center of the metalens (see figure below). This type of metalens can be used for focusing incoming parallel rays to a point. Unlike normal lenses, the metalens will look just like an ultrathin circular disc.



Consider two homogeneous media of refractive indices n_1 and n_2 separated by a metalens as shown in the figure below. Suppose a beam of plane wave is incident at point Q (at distance r from the pole P) on the metasurface at an angle of θ_1 from medium 1. Assume that the rays falling at Q are in the plane containing PQ and the axis of the lens (x-z plane). These rays will be refracted at an angle θ_2 in the medium 2. The angle of refraction θ_2 depends on r. Thus, the modified law of refraction for the metasurface can be written as

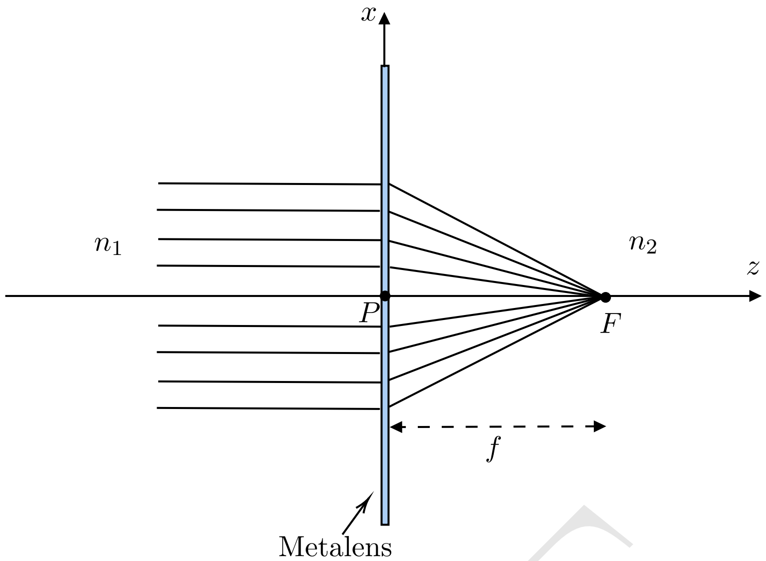
$$n_1 \sin\theta_1 - n_2 \sin\theta_2 = f(r)$$

Similarly, the rays falling at Q', at a distance r', will be refracted by an angle θ_2'.

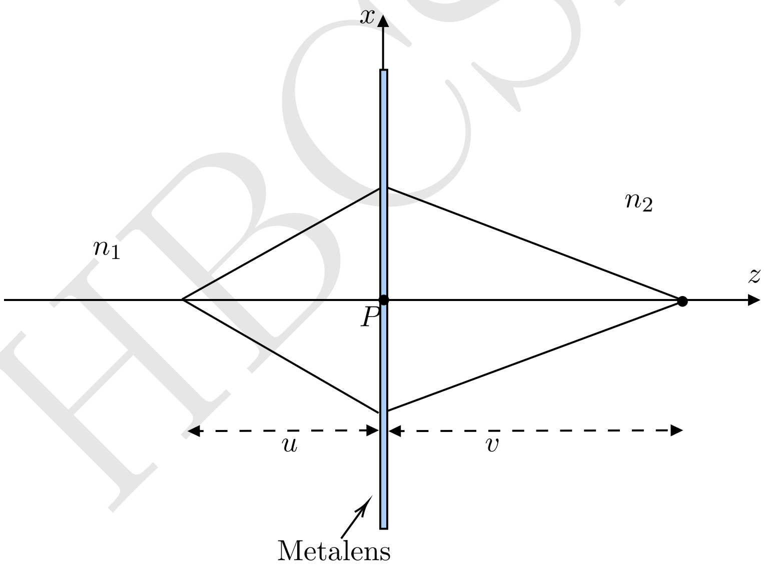


(a) **[8 marks]** Find $f(r)$ in terms of $\phi(r)$ and k_o, the wave number of the incoming wave in a vacuum. To determine $f(r)$, assume two rays in x-z plane incident at an angle θ_1 at two infinitesimally close points, r and $r + \Delta r$, are refracted by the same angle θ_2. You don't need to derive the exact functional form of $\phi(r)$ for this part.

(b) [**4 marks**] Derive an expression for the phase profile $\phi(r)$, to convert a plane wavefront to spherical wavefront, with light being focused to a point F on the axis (see figure below), which is at a distance f from the pole P.



(c) [**3 marks**] Consider a metalens whose phase profile is obtained in part (b). For the paraxial approximation, derive an expression for the lens equation, having object distance u and image distance being v, with focal length f (see figure below).



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