Indian National Astronomy Olympiad (INAO) - 2025

Jointly organized by
Homi Bhabha Centre for Science Education (HBCSE-TIFR)
and
Indian Institute of Science Education and Research Mohali (IISER-Mohali)

Question Paper

Roll Number: ☐☐☐☐ ☐☐☐☐ ☐☐☐☐ **Date:** 01 February 2025

Duration: Three Hours **Maximum Marks:** 100

Please Note:

- Write your **Roll Number** in the boxes above.
- There are a total of **10 pages and 12 questions** in this booklet. Maximum marks for each sub-question are indicated.
- For all questions, the *process* involved in arriving at the solution *is equally important* as the final answer. Valid assumptions and approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.

Useful Constants

Quantity	**Symbol**	**Value**
Mass of the Sun	$M_\odot$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	5.972×10^{24} kg
Mass of the Moon	M_c	7.347×10^{22} kg
Radius of the Sun	$R_\odot$	6.955×10^{8} m
Radius of the Earth	$R_\oplus$	6.371×10^{6} m
Radius of the Moon	R_c	1.737×10^{6} m
Speed of Light	c	2.998×10^{8} m/s
Astronomical Unit	$a_\oplus \equiv 1$ A.U.	1.496×10^{11} m
Sun-Venus distance	0.72 A.U.	1.077×10^{11} m
Solar Constant (at Earth)	S	1366 W/m^2
Gravitational Constant	G	6.674×10^{-11} N m^2/kg^2
Planck Constant	h	6.626×10^{-34} J/Hz

IISER Mohali,
Knowledge City, Sector 81,
S.A.S Nagar, Punjab - 140306, India

Question 1

Consider a telescope with a primary (convex lens) with a diameter of 5 cm and a focal length of 60 cm, and a secondary (convex lens) with a focal length of 3 cm. This telescope is used to project a perfectly focused image of the Sun at a distance of 60 cm from the secondary. You are given that the diameter of the Sun subtends an angle of $30'$ in the sky.

(a) What is the distance between the primary and secondary? [2]

(b) Draw a ray diagram marking all the optical elements, images and distances. [3]

(c) What is the size of the image? [3]

(d) What is the physical size of the smallest sunspot that can be resolved by this arrangement? [2]

Question 2

An isolated star outside the solar system normally moves with a certain constant velocity with respect to the Sun. However, if there is a planet in orbit around it, there will be an appropriate periodic change in its motion. This variation in motion can now be detected using sensitive measurements of the Doppler shift of spectral lines. However, only the line-of-sight component of the total velocity of the star can be detected using this method. This motion, combined with the time period of the orbital motion, can be used to estimate the mass of a planet orbiting a distant star.

Suppose that a star of mass m_s has a planet of mass m_p in a circular orbit. The separation between the objects is r, and circular speed of the planet is v_p.

(a) Assuming that $m_p \ll m_s$, find an expression for m_p in terms of observables (orbital time period P, v_s : the periodic variation in the radial velocity of the star, and mass of the star m_s that can be estimated from other considerations). [3]

(b) We can determine the time period P from the periodicity of variation of Doppler shift in the spectral lines and estimate m_s from the spectral type of the star. A special case is where the planet is transiting in front of (i.e. partially eclipsing) the star as it orbits. In this case the inclination is known to be 90°. This leads to the light curve of the type shown in Figure 1. Compute the orbital radius of the orbit of the planet around the star. State your approach and reasoning clearly. You are provided that $P = 6.1$ days, $m_s = 0.0898\ M_\odot$ and $v_s = 1.353\ m/s$. Compute the mass and the radius of the planet in the system for which the light curve has been given in Figure 1. [5]

(c) Albedo is defined as the reflectivity of a planet. Earth has an Albedo of 0.31, for instance. Assume the planet under consideration has a similar Albedo to that of the Earth. Determine the average surface temperature of the planet, and thus whether water can be found on it in the liquid state, an important condition for a planet to be habitable. The radius and the temperature of the star are $0.1192\ R_\odot$ and $T_s = 2566$ K respectively. [5]

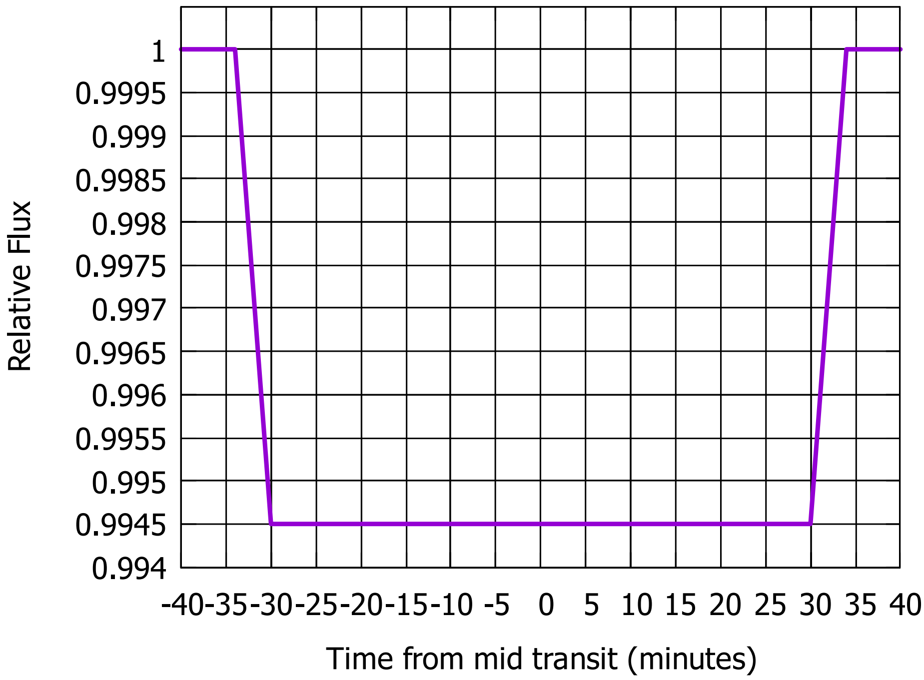


Figure 1: Light curve of the Planet.

(d) Planets with average density above 3 g cm^{-3} can be classified as rocky while those below this density can be dubbed as gaseous planet. Find out if the planet is rocky or gaseous. [2]

Question 3

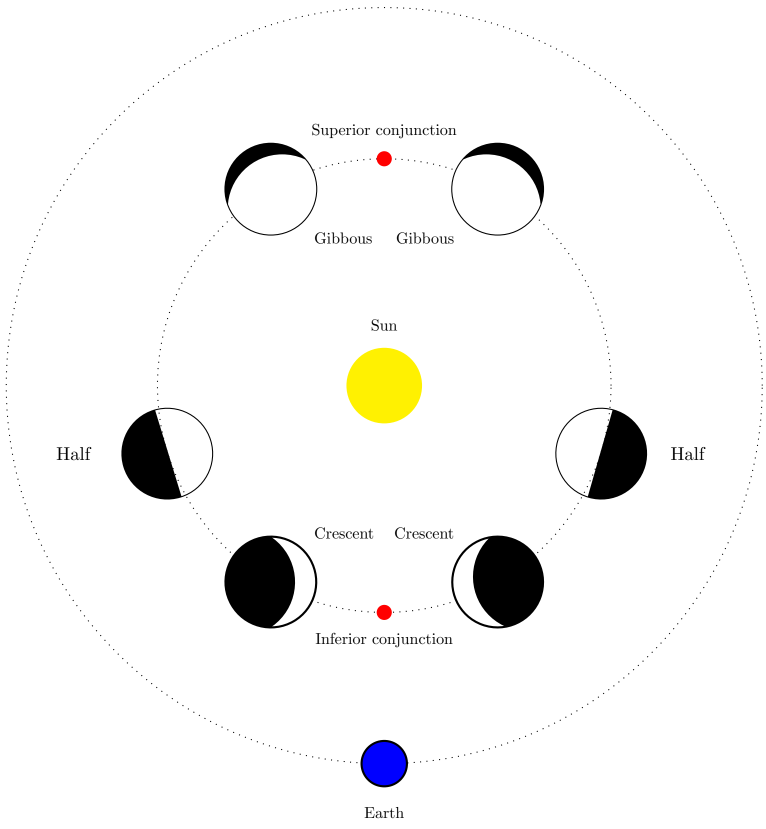
The flux of light reaching the earth from Venus varies due to (i) the distance of Venus from Sun, (ii) the distance of Venus from the observer on the Earth, (iii) the amount of illuminated surface visible to the observer and (iv) the reflective power of the Venus surface and its rotational speed (see Figure below).

(a) Set up the equation for flux of light reaching the Earth after reflection from Venus. Assume the Sun-Venus and the Sun-Earth orbit to be circular in the same plane with the radii r and Δ respectively; the radius of Venus to be a, the Earth-Venus distance to be ρ and the Earth-Venus-Sun angle to be ψ. [5]

(b) Find the value of ρ (in terms of r and Δ) so that Venus has maximum flux at that distance. [5]

(c) Estimate the portion of the Venus disk that is illuminated at its maximum flux. Is it in Crescent or Gibbous phase? [2]

(d) The flux received from Venus varies by a factor of 3 between extremes during its entire synodic cycle. How do you explain this almost constant apparent flux in spite of large variation in distance and illumination? [3]



Phases of Venus

(Figures not to scale)

Question 4

In a planetary system similar to ours (see Figure 2),

- A planet P has a moon M, and the P-M system orbits their star S.
- The orbit of M around P and that of the P-M system around S are circular, coplanar and in the same sense of rotation.
- The radius of P has been measured to be 6000 km.
- Laser ranging has determined the distance between P and M to be 400,000 km and that between S and the P-M system to be 150×10^6 km respectively.
- The angular size of S viewed from the centre of mass of the P-M system and that of M viewed from P are $30'$ and $31'$ respectively.
- M completes one orbit around P in 30 Earth days and orbital period of P around S is 360 Earth days.
- Planet P spins around its axis once in 25 Earth hours. The spin of M is synchronised with its orbit around P. The sense of rotation of P and M and of their revolution in their respective orbits are the same.

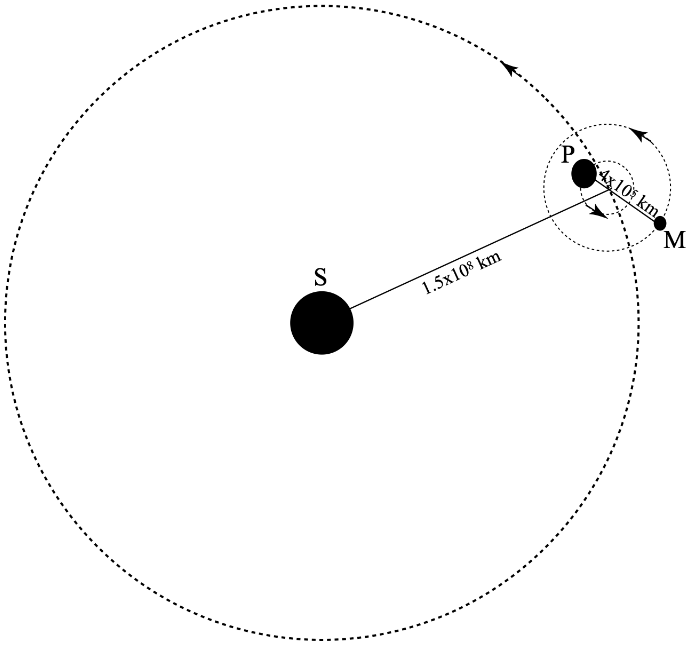


Figure 2: A schematic sketch of the Star S - Planet P - Moon M system

Given the above:

(a) Find the mass of S. [2]

(b) Assuming P and M are made of the same material, and their density is uniform, find the masses of P and M. [3]

(c) Are all the eclipses of S viewed from the equator of P Total? If so, what is the duration of totality? [3]

(d) Fossil records suggest that 1×10^9 Earth years ago the spin rate of P was 22 Earth hours. Tidal interaction has caused angular momentum to be transferred from the spin of P to the P-M system. What was the orbital period of M around P, 1×10^9 years ago? [3]

(e) If tidal angular momentum transfer continues at the same rate, when will the eclipses of S by M as viewed from the equator of P cease to be total? What will be the orbital period of M and spin period of P then? [4]

Question 5

The axes of the Hertzsprung–Russell (HR) diagram in Figure 3 represent two fundamental observable properties of stars: luminosity L (the amount of energy radiated per unit time or how bright a star is) on the y-axis and surface temperature (how hot it is on the outside, which is the only part we can directly see) on the x-axis. Typically, the luminosity is expressed in units of the Solar luminosity ($L_{\text{sun}} = 3.826 \times 10^{26}$ W). The surface temperature of the Sun is approximately 5770 K. The most common form of the H–R diagram uses an unusual convention by representing the direction of increasing temperature values backwards compared to most graphs: temperature increases to the left and decreases to the right. Each square and pentagonal symbol in Figure 3 represents a star. The majority of stars lie on the main sequence (the cloud of points spread diagonally in the centre). You can assume that stars are in equilibrium for most of their lifetime. Note that the temperatures have been plotted on a logarithmic scale (base 10) while the logarithm (base 10) of the luminosity (in units of solar luminosities) have been used.

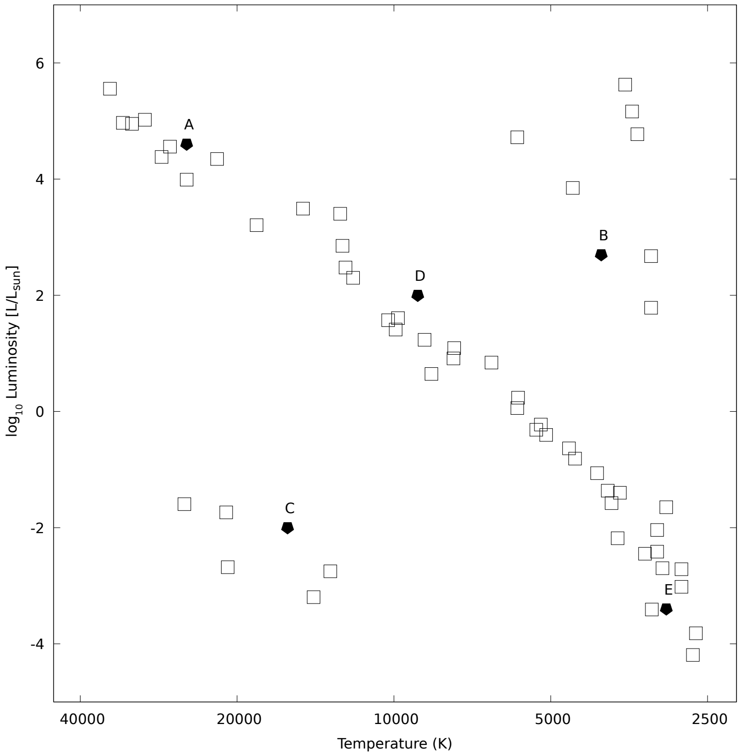


Figure 3: The HR diagram

Another property responsible for determining other characteristics of main-sequence stars is its mass M. Stars with more mass have stronger gravity, and therefore achieve higher core temperatures. Since fusion occurs only in the core, higher core temperatures produce higher fusion reaction rates, and this leads to higher luminosity. The lifetime of a star is usually defined as the amount of time that star spends fusing hydrogen into helium in its core, which is the time the star spends on the main sequence. That time is determined by the amount of hydrogen fuel initially in the star's core (which is proportional to its mass) and the rate at which the star uses that fuel. For main sequence stars $L \propto M^{3.5}$.

(a) Order the radii (increasing order) of stars labeled A, B, C, D, E (in Figure 3 and Table 1) [5]

(b) For main sequence stars, A and E, find their lifetime relative to the lifetime of the Sun. [2]

Table 1: Properties of Stars

Star Label	Temperature (K)	Luminosity ($L_{\text{star}}/L_{\text{sun}}$)
A	25000	40000
B	4000	500
C	16000	0.01
D	9000	100
E	3000	0.0004

Question 6

Refer to the attached Figure 4. Three identical gas clouds are marked as A, B, and C. Each gas cloud emits EM radiation of luminosity L at a spectral line having the rest-frame frequency F. The gas clouds A, B, and C are in circular motion, in the counterclockwise direction, with identical speed V, in three different co-centric circles with radius R, 2R, and 3R respectively. A Telescope is situated at the point T in the circle having radius 2R, also in circular motion with speed V in the counterclockwise direction. The center of the circles is marked as O. At an instance, $\angle TAO = 90°$ and the segment CTOB forms a straight line.

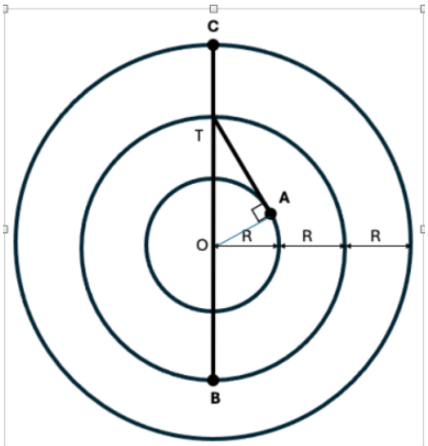


Figure 4: Locations of gas clouds

Answer the following:

1. Estimate the magnitude of line-of-sight velocity of the gas cloud A, as seen by the telescope T. [4]

2. Estimate the ratio of flux received at T from gas clouds A, B, and C. [2]

3. Which cloud(s) will show maximum Doppler frequency shift in the spectral line signal received at T. Explain your answer. [2]

Question 7

A spacecraft is in a circular orbit around the Sun with an orbital radius of 1 A.U. The spacecraft deploys a panel of area A with the normal of the panel pointing towards the Sun. The panel is made of a perfectly reflecting material. What can be the maximum mass M of the spacecraft **(including that of the panel)** such that the spacecraft escapes the solar system? [5]

Question 8

Let us consider a distant point-like object, say a star, which is emitting radiation isotropically. The star is surrounded by numerous hydrogen clouds, uniformly distributed over a large radius r. The radiation from the star after getting scattered from the gas clouds arrive as parallel rays to a distant observer who collects these with a lens, as depicted in Figure 5. Derive the expression for the locus of clouds that will result in a fixed time delay of $\tau = R/c$ (known as an isodelay surface) to echo the brightness change of the star? For simplicity, consider a cloud distribution in 2-dimensions with the star at $(x, y) = (0, 0)$. [5]

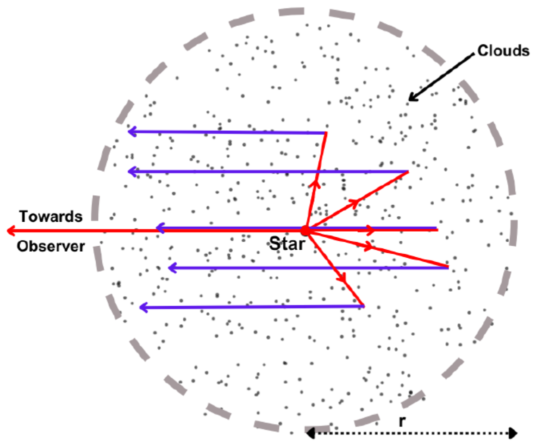


Figure 5: Star is in the centre and all the dots are the clouds absorbing radiation incident on them from the star and then emitted as line radiation towards a distant observer.

Question 9

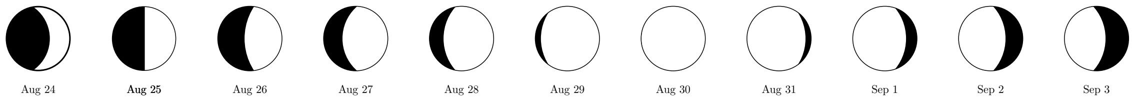
On August 16, 2023, it was new Moon and on August 23, 2023, 6 PM IST (the Chandrayan-3 landing event) it was sixth day of the waxing moon. Nearly 30% of Moon was illuminated as illustrated below:

The orbital plane of Moon is tilted by about 5°, with respect to the ecliptic plane, and it orbits in an elliptical orbit. However, assume that the path of Moon around the Earth is coplanar with ecliptic and the orbit is circular.

An observer from Moon (nearside close to equator) will see the illuminated portion of the Earth change from August 23, 2023 onward as illustrated in ______ (fill (a), (b), (c) or (d)) and explain your choice (Ignore the difference in the size, if any.). [5]

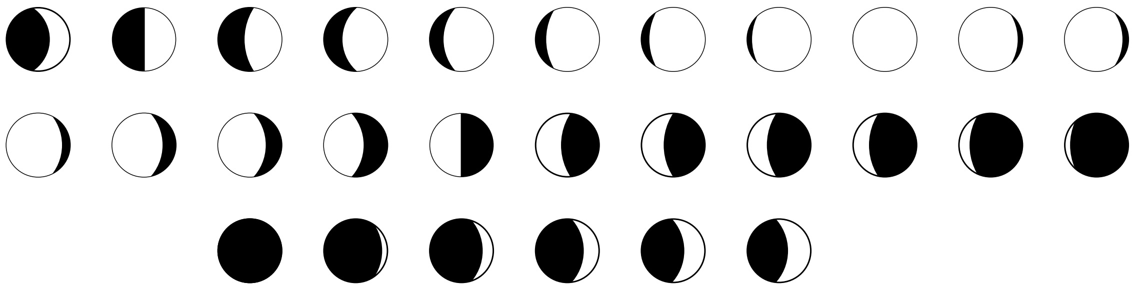
(a)

(b)



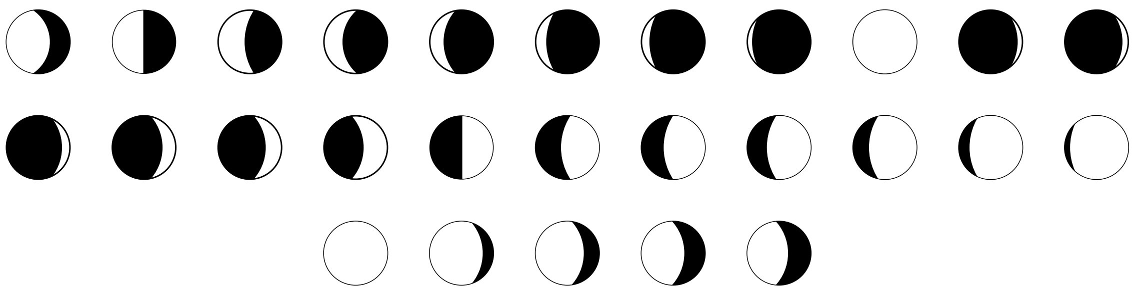
(c)

Every 24 hours earth will undergo a cycle of full phases, starting from waxing cresent, at the time of landing.



(d)

Every 24 hours earth will undergo a cycle of full phases, starting from waning cresent, at the time of landing.



Question 10

Consider Earth as a sphere of radius 6400 km. At the time of equinox (i.e. center of the Sun's disk is at declination 0.0 deg), an observer is standing exactly at North pole with a drone (a flying device) camera. The observer wishes to see the full disk of the Sun, which subtends an angle of 0.009 radian to the observer. At what minimum height, flying above the North pole, the drone camera can see the full disk of the Sun. Assume that the local horizon is clear without any geographical obstructions (land, trees, building etc.) to the horizon and sun-rays are parallel. Also ignore all light-ray bending effects such as atmospheric refraction etc. [5]

Question 11

A powerful luminous source is orbiting almost touching to the surface (at equator) of a spherical white dwarf in a circular orbit. The rotation orbit is perpendicular to the line joining star and us. Variation of light received from this source has A periodicity of 8 seconds. What is the density of white dwarf? [5]

Question 12

A photon leaving the surface of non-spinning compact stars like white dwarf or Neutron star will face intense gravity, leading to a change in the wave energy E associated with the photon (this phenomenon is generally called gravitational redshift). For gravity effect on photons, an effective mass m can be assumed through the relation $E = mc^2$. Estimate the fractional change in the frequency of a photon leaving the surface of a white dwarf, Sirius B of mass M=1$M_\odot$ and radius $R = 0.008R_\odot$, when observed by a distant observer (assuming Newton's gravity). [5]