Indian National Astronomy Olympiad (INAO) – 2024

Question Paper

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ Date: 03 February 2024

Duration: **Three Hours** Maximum Marks: 100

Please Note:

- Before starting, please ensure that you have received a copy of the question paper containing total 5 pages.
- Please write your roll number in the space provided above.
- There are total 5 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_{☾}$	$\approx$	7.347×10^{22} kg
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Radius of the Moon	$R_{☾}$	$\approx$	1.737×10^{6} m
Speed of Light	c	$\approx$	2.998×10^{8} m/s
Astronomical Unit	$a_\oplus$	$\approx$	1.496×10^{11} m
Solar Constant (at Earth)	S	$\approx$	$1366\,\mathrm{W/m^2}$
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$

1. Answer each of the following sub-questions with a brief justification -

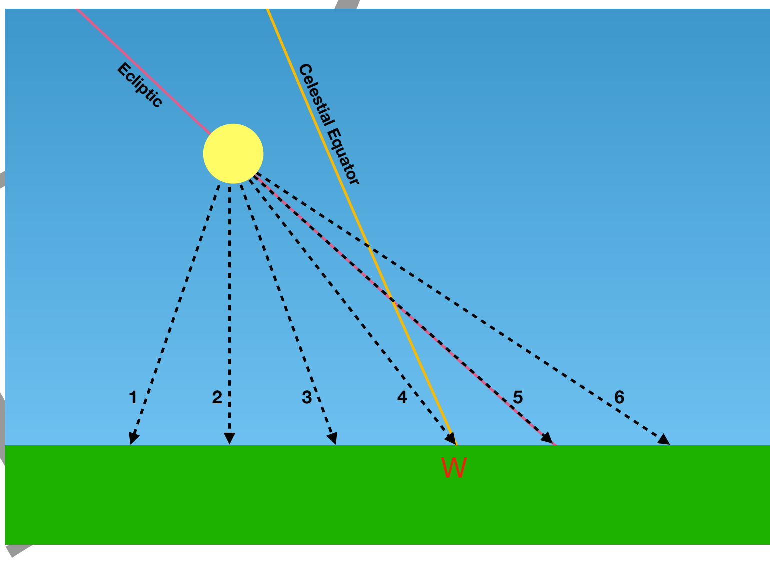
(a) (2 marks) Fatima observed a celestial object, with her unaided eyes, very near to the western horizon immediately after sunset on 17 July 2023. She continued to observe the same object at the same time everyday till 4 September 2023. She made the following observations-

- Object's separation from the Sun increases starting from 17 July.
- Object's separation from the Sun decreases after 11 August.
- She also made an observation that the object was never seen overhead at sunset within that year.

Name one object, which is not a comet, that fits these observations made by Fatima.

Solution:
Mercury /Venus - inner planet.

(b) (2 marks) In the image below, you see the Sun just above the western horizon on some day as seen from the city of Ujjain. The ecliptic, celestial equator and due west point (W) are marked in the image. Which of the paths marked (1 to 6) on the image closely represents the path that Sun will follow on its way to the horizon on that day.



Solution:
The Sun's daily path is almost parallel to the celestial equator. Hence, path number 3 is the correct answer.

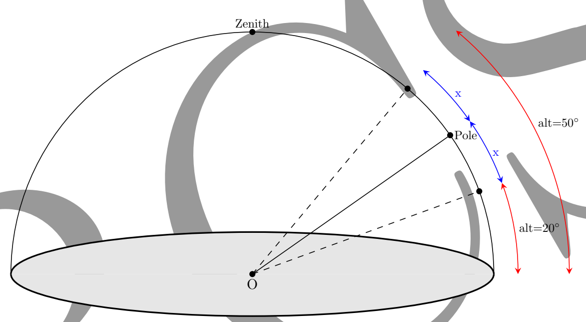
(c) (2 marks) Three observers A, B and C are stationed on Moon, Venus and Mars respectively. Which of these observer(s) can see almost all the phases of Earth [new (no earth), crescent, half, gibbous, etc.].

Solution:

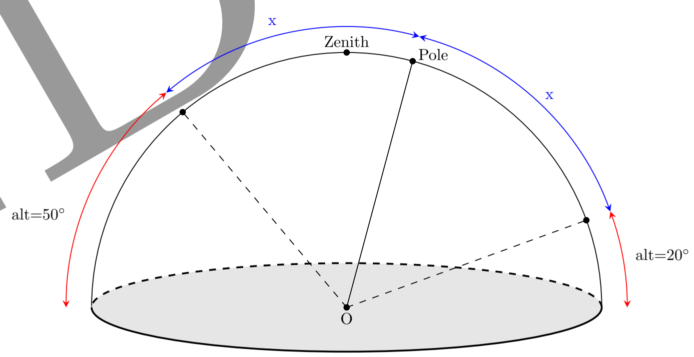
Observer A and C will see almost all the phases of the Earth.
For Moon, as Earth based observer sees lunar phases, similarly, a Moon based observer will see the phases of Earth.
For Venus based observer, Earth is an outer planet, hence, the observer will never see a phase of Earth where most of the Earth is dark.
For Mars based observer Earth is an inner planet and the observer will see all the phases including nearly full Earth.

(d) (4 marks) A certain star was observed to remain always above horizon over the course of one full day. During this period its maximum altitude was 50° and minimum altitude was 20°. What is/are the latitude/s for the place of observation?
Note: Altitude is the angular distance of a star from the horizon measured along a circular arc perpendicular to the horizon.

Solution:
If maximum and minimum altitude are on same side of Zenith = ±35°.



If maximum and minimum altitude are on either side of Zenith = ±75°.



(e) (2 marks) In which month/s of the year, will there be a New Moon in the constellation of Leo?
A. August B. November C. February D. May

Solution:
Option (A) - August
Sun is in the constellation of Leo in the month of August.
New moon will occur in the same constellation.

(f) (4 marks) Which of the following statement(s) CANNOT be inferred from Kepler's laws of motion?

A. A planet moves in an elliptical orbit around the Sun.

B. The eccentricity of the orbits of all solar system planets is small.

C. A solar system planet has its highest tangential velocity when it is closest to the Sun.

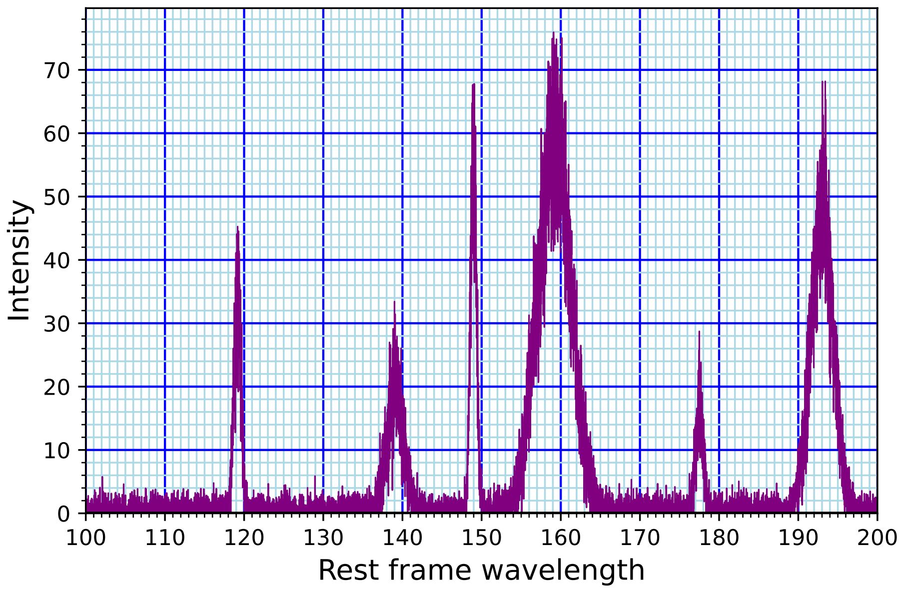
D. All the planets move in elliptical orbits in roughly the same plane around the Sun.

Solution:

B - Keplers laws doesn't make any statement about the value of the eccentricity.

D - Keplers laws doesn't make any statement about the orientation of the plane.

(g) (5 marks) The presence of different elements in a distant astronomical object is inferred from the peaks in the emission spectrum of its soil, at certain wavelengths, characteristic to each element. One such spectrum for a moon (named $Soma314-b-1$) around a distant exoplanet observed by the hypothetical mission Chandrayaan−300, that landed on the surface of $Soma314-b-1$ in the future is shown below. The spectrometer is noisy i.e., small random fluctuations get superimposed on top of the actual signal. The units of both the axes in the figure below are arbitrary.



A complete table of emission peaks (in same arbitrary length unit as in the above figure) of various hypothetical elements is given below. It is assumed that the strength of all the peaks indicated in the table is sufficiently high to be observed in the spectrum shown above if the corresponding element is present in the source.

Identify the elements present on the surface of $Soma314-b-1$.

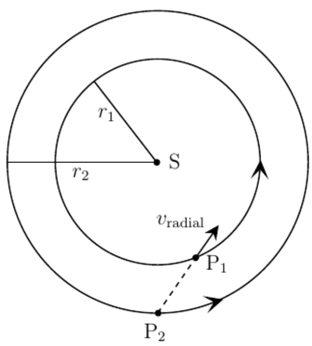
Element	**Wavelength 1**	**Wavelength 2**	**Wavelength 3**
Fh	193.44	–	–
Dz	149.18	159.73	–
Ab	111.71	122.87	177.94
Hm	132.67	139.56	–
Cw	119.55	139.32	–
Xy	148.90	159.69	–

Table 1: Emission lines

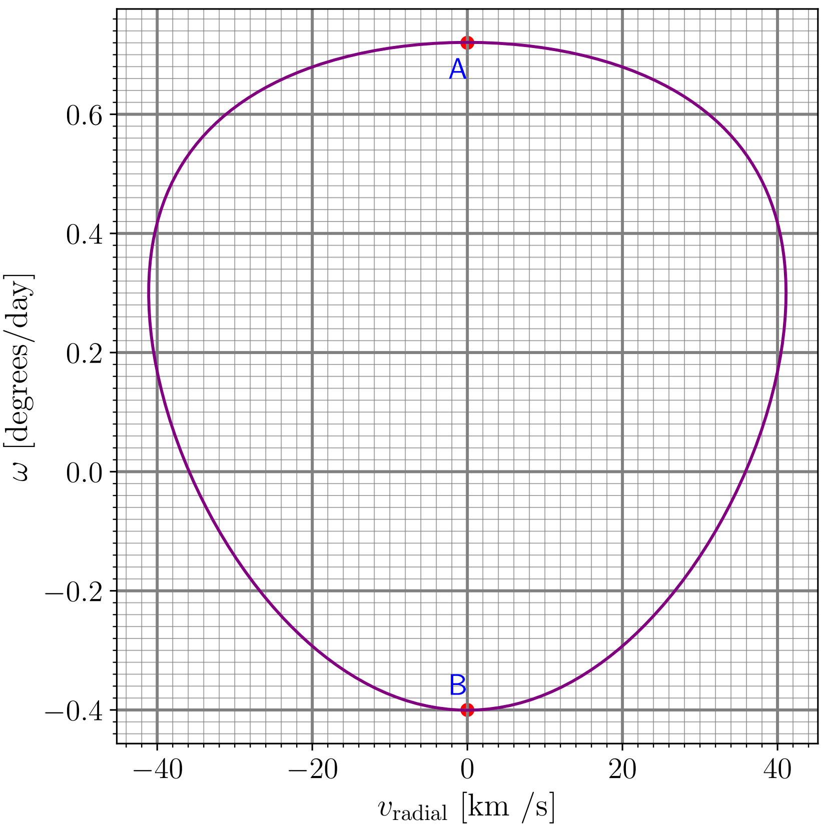
Solution:

Element	**Present/Absent/Maybe**	**Reason**
Fh	Present	Peak near 193 nm is distinct from other element and clearly visible.
Dz	Maybe	Since it cannot be distinguised from Xy.
Ab	Absent	Two lines are absent.
Hm	Absent	Peak near 132 is clearly absent.
Cw	Present	Although peak near 139 is not distinguishable from Hm, presence of peak near 119 resolves the ambiguity.
Xy	Maybe	Can not be distinguised properly from Dz.

2. Consider a system of two planets P_1 and P_2, as shown below, both revolving in the same direction in circular coplanar orbits of radii r_1 and r_2, respectively, around a star S of mass M much larger than the masses of the planets. Anilesh is stranded on the outer planet P_2. Both the star S and the planet P_1 are seen to move against the stellar background by Anilesh due to the orbital motions of P_1 and P_2. Taking the direction of motion of S as positive, he measures the angular velocity ω of P_1 against the stellar background. He also measures the velocity of P_1 along the line of sight (called the "radial velocity", v_{radial} in astronomy) as shown in the figure below.

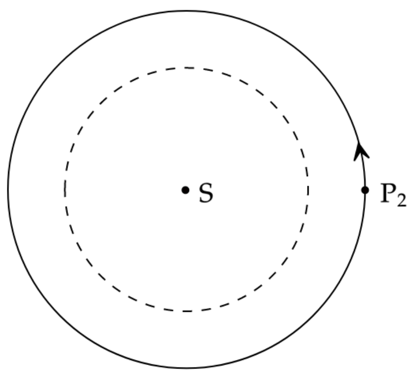


He obtained the following curve for the variation of the ω (in degrees/day, 1 day being 24 hours) versus v_{radial} (in km/s).

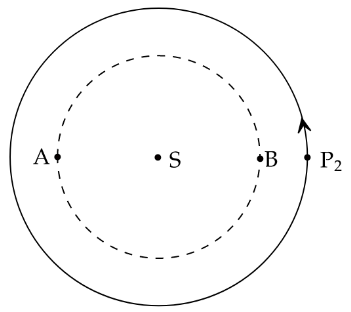


Plot of ω vs v_{radial}

(a) (3 marks) For the given position of P_2 as marked in the following figure; show the positions of P_1 corresponding to the points A and B on the graph above by marking appropriately on the orbit of P_1 (dashed circle in the figure in your answersheet).



Solution:



(b) (7 marks) Find the ratio r_1/r_2.

Solution:

The maxima and minima of angular velocity ω is attained at the points corresponding to $v_{\text{radial}} = 0$.

This happens at either superior conjunction (say point **A**) or inferior conjunction (**B**). Note that, by the definition of ω in the question, $\omega > 0$ at superior conjunction. Let r_1 and r_2 be the radii of P_1 and P_2 respectively and ratio of inner planet radius to outer planet radius be $x = \frac{r_1}{r_2}$. Also let ω_A and ω_B be the angular velocities at superior and inferior conjunctions respectively. We have

$$|\omega_A| = \frac{v_1 + v_2}{r_1 + r_2} = \frac{\sqrt{\frac{GM}{r_1}} + \sqrt{\frac{GM}{r_2}}}{r_1 + r_2} = \sqrt{\frac{GM}{r_2^3}}\frac{1 + \frac{1}{\sqrt{x}}}{1 + x}$$

$$|\omega_B| = \frac{v_1 - v_2}{r_2 - r_1} = \frac{\sqrt{\frac{GM}{r_1}} - \sqrt{\frac{GM}{r_2}}}{r_2 - r_1} = \sqrt{\frac{GM}{r_2^3}}\frac{\frac{1}{\sqrt{x}} - 1}{1 - x}$$

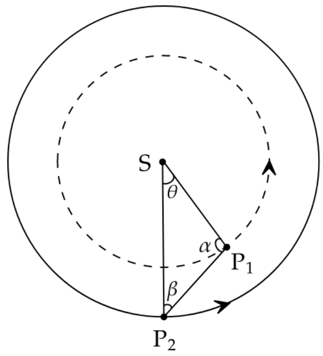
$$\frac{|\omega_A|}{|\omega_B|} = \frac{1 + x + 2\sqrt{x}}{1 + x}$$

From the graph we see that $\omega_A = 0.72^\circ$ day^{-1} and $\omega_B = -0.40$ degrees day^{-1}

$$\frac{1 + x + 2\sqrt{x}}{1 + x} = 1.8 \implies x = 0.25 \text{ or } 4$$

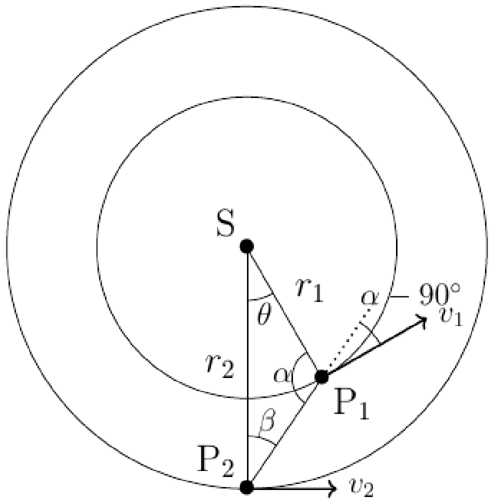
Since $r_1 < r_2$ we consider $\boxed{\frac{r_1}{r_2} = 0.25}$

(c) (8 marks) Write the expression for $v(r)$ as a function of β (see figure below). Calculate the value of θ for which v_{radial} is maximum.



Hint: You may find it useful to use the sine rule here.

Solution:
Let v_1 and v_2 be the orbital velocities of P_1 and P_2 respectively.



$$v_{\text{radial}} = [v_1 |\cos(\alpha - 90^\circ)|] - [v_2 |\sin\beta|]$$
$$v_{\text{radial}} = [v_1 |\sin\alpha|] - [v_2 |\sin\beta|]$$

Using sine rule in triangle P_1SP_2, we get that

$$\frac{r_1}{|\sin\beta|} = \frac{r_2}{|\sin\alpha|}$$
$$|\sin\alpha| = \frac{r_2}{r_1}|\sin\beta|$$

Which implies that

$$v_{\text{radial}} = \left(v_1 \frac{r_2}{r_1} - v_2\right) |\sin\beta|$$
$$= v_2 \left[\frac{v_1}{v_2} . \frac{r_2}{r_1} - 1\right] |\sin\beta|$$
$$= v_2 \left[\left(\frac{r_2}{r_1}\right)^{3/2} - 1\right] |\sin\beta|$$

Hence, maximum value of v_{radial} occurs when β is maximum and $\alpha = 90°$, which is also called **maximum elongation**.
Therefore, maximum value of v_{radial} occurs at angle P_2-S-P_1 or

$$\boxed{\theta = \cos^{-1}\left(\frac{r_1}{r_2}\right) \approx 75.5°}$$

(d) (8 marks) Determine M, r_1 and r_2.

Solution:
The maximum value of v_{radial} occurs when $\alpha = 90°$ is,

$$(v_{\text{radial}})_{\max} = v_1 - v_2 \frac{r_1}{r_2}$$
$$= \sqrt{\frac{GM}{r_1}} - \frac{r_1}{r_2}\sqrt{\frac{GM}{r_2}}$$

From the graph, $(v_{\text{radial}})_{\max} = 42$ km s^{-1}

Accept 41 km/s as well.

and from previous part we know, $x = \frac{r_1}{r_2} = 0.25$

$$42000 = \sqrt{\frac{GM}{r_2}}\left(\frac{1}{\sqrt{x}} - x\right) = 1.75\sqrt{\frac{GM}{r_2}}$$

Also, reading the value of ω_A

$$\omega_A = 0.72 \text{ degrees day}^{-1}$$
$$\omega_A = 1.454 \times 10^{-7} \text{rad s}^{-1}$$
$$\therefore 1.454 \times 10^{-7} = \sqrt{\frac{GM}{r_2^3}} \frac{1 + x^{-1/2}}{1 + x} = 2.4\sqrt{\frac{GM}{r_2^3}}$$

Solving the above two equations, we get that

$$\boxed{r_2 = 3.96 \times 10^{11} \text{ m } = 2.65 \text{ AU}}$$
$$\boxed{r_1 = 0.25\ r_2 = 9.90 \times 10^{10} \text{ m } = 0.66 \text{ AU}}$$
$$\boxed{M = 3.42 \times 10^{30} \text{ kg } = 1.7 M_\odot}$$

3. Consider two containers A and B both with the same amount of liquid of volume V. Initially, A only has milk and B only has water. We transfer liquids back and forth between the two containers. One transfer is defined as completion of both the following steps:

- **Step 1**: Take some fixed volume L from container A, put it in B and mix it well.
- **Step 2**: Take the same volume L of the mixed fluid from container B and put it back in A and mix it well.

At the end of each transfer, both containers have liquids with exactly the same volume V. Let $C_M^A(n)$ denote the concentration of milk in container A at the *end* of the n-th transfer. Here we define concentration as

$$C_M^A(n) = \frac{\text{Volume of milk in container } A}{\text{Total Volume in container } A} \text{ after } n \text{ complete transfers}$$

Similarly, you may define - C_W^A, C_M^B and C_W^B.

(a) (4 marks) Write down the expression for the concentration of milk in container A at the end of the *first* transfer. Express the concentration $C_M^A(1)$, after the *first* transfer, in terms of volumes L and V.

Solution:

- At the very beginning, the concentration of milk in A is $C_M^A(0) = 1$.
- Volume of milk transferred from A to B
  $=$ concentration in $A \times$ volume transferred $= C_M^A(0) \times L = L$
- Total volume of milk in B after Step 1 $= L + 0$
- Total volume in B after Step 1 $= V + L$
- Concentration of milk in B after Step 1 be $C_M^B = \dfrac{L}{(V+L)}$
- Volume of milk remaining in container A after Step 1 is $V - L$
- Volume L is transferred back to A from B
- Amount of milk transferred back to container A

$$= \text{concentration of milk in } B \times \text{volume transferred}$$
$$= C_M^B \times L$$

- Total volume of milk in container A at the end of the first transfer

$$=(V-L)+C_M^B \times L$$

- Concentration of milk in container A at the end of the first transfer

$$=\frac{\text{Total volume of milk}}{\text{Total Volume of Container } A}$$

- Hence,

$$C_M^A(1) = \frac{(V-L)+C_M^B \times L}{V}$$
$$C_M^A(1) = \frac{(V-L)+\frac{L}{V+L} \times L}{V}$$
$$C_M^A(1) = \frac{V}{V+L}$$

(b) (8 marks) Let us define ϵ as, $\epsilon = \dfrac{L}{V}$. Write down the recursion relation for $C_M^A(n)$, in terms of the concentration at the end of the $(n-1)$th transfer *i.e.,* $C_M^A(n-1)$ and ϵ.

Solution:
Let $C_M^A(n)$ and $C_W^A(n)$ be the concentration of milk and water in container A resp. and $C_M^B(n), C_W^B(n)$ be the concentration of milk and water in container B at the end of the n-th transfer. It is clear that from conservation of volume in each container, we have

$$C_M^A(n) + C_W^A(n) = 1 \quad \text{and} \quad C_M^B(n) + C_W^B(n) = 1$$

Now, also note that the total volume of milk across containers A and B together has to be equal to V (what we started with) and similarly for water. Hence,

$$C_M^A(n) + C_M^B(n) = 1 \quad \text{and} \quad C_W^A(n) + C_W^B(n) = 1$$

Hence, the problem can be completely described by just one concentration -

$$C_M^A(n) = c_n$$

i.e., concentration of milk in A after n transfers.Transfer Process:

- At the start of the transfer the concentration of milk in A is c_{n-1}
- Volume of milk transferred from A to B in Step 1
  = concentration in A × Volume transferred
  $= c_{n-1} \times L$
- Volume of milk left behind in A after Step 1
  $= c_{n-1} \times (V-L)$

- Volume of milk in B before Step 1
$= C_M^B(n-1) \times V = (1 - c_{n-1})V$

- Total volume of milk in B after Step 1
$= (1 - c_{n-1}) \times V + c_{n-1} \times L$

- Concentration of milk in B after Step 1
$C_M^B = \dfrac{(1 - c_{n-1})V + c_{n-1} \times L}{V + L}$

- Volume L of fluid is now transferred back to A

- Amount of milk in volume L from container B
$C_M^B \times L = \dfrac{(1 - c_{n-1})V + c_{n-1} \times L}{V + L} \times L$

- Total amount of milk in A after the transfer is complete
$= [c_{n-1} \times (V - L)] + [C_M^B \times L]$

- Concentration of milk in A after the transfer is complete
$c_n = \dfrac{[c_{n-1} \times (V - L)] + \left[C_M^B \times L\right]}{V}$

- Substituting for C_M^B gives

$$c_n = c_{n-1}\left(\frac{V - L}{V + L}\right) + \frac{L}{V + L}$$

- Substituting for $L = \epsilon V$, gives

$$C_M^A(n) = C_M^A(n-1)\left(\frac{1 - \epsilon}{1 + \epsilon}\right) + \frac{\epsilon}{1 + \epsilon}$$

(c) (3 marks) If one does fairly large number of transfers, then the concentration of milk in either of the container will reach almost equilibrium value which is a function of ϵ. What will be the value of this equilibrium concentration?

Solution:
At equilibrium, concentration in both the containers must be equal as $C_M^A + C_M^B = 1$, it means concentration in either of the containers should be 1/2.
<u>Alternate solution</u>.
The recursion relation is a map between the concentration in the $(n-1)^{\text{th}}$ and n^{th} transfer.
After equilibrium is reached, the concentration will be the same regardless of any number of transfers. Let us say that equilibrium is reached at z-th transfer. Therefore, we can say $c(z) = c(z + 1)$.
Solving using this condition we get $c(z) = 1/2$.

(d) (5 marks) Define a new variable f_n such that,

$$f_n = C_M^A(n) - 1/2$$

and simplify the recursion relation between f_{n+1} and f_n to express $C_M^A(n)$ as a function of ϵ and n.

Solution:
Substitute, $f_n = C_M^A(n) - 1/2$
With this the recursion relation becomes

$$f_{n+1} = f_n \left(\frac{1-\epsilon}{1+\epsilon}\right)$$

and we get $f_n = f_0 \left(\frac{1-\epsilon}{1+\epsilon}\right)^n$
Since, $C_M^A(0) = 1$, $f_0 = 1/2$

$$f_n = \frac{1}{2}\left(\frac{1-\epsilon}{1+\epsilon}\right)^n$$

Since, $f_0 = 1/2$ then $C_M^A(n)$ is

$$C_M^A(n) = \frac{1}{2}\left(\frac{1-\epsilon}{1+\epsilon}\right)^n + \frac{1}{2}$$

4. The difference in magnitudes m_1 and m_2 of two celestial objects with flux (energy received from the object per unit area per unit time) F_1 and F_2, respectively, is given by Pogson's relation as

$$\Delta m = m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

(a) (3 marks) Calculate the total energy per unit time received at the primary mirror of diameter 6 inches (1 inch = 2.54 cm) of a telescope from the star Sirius (magnitude $m_{\text{sirius}} = -1.46$). The flux of the star Vega, which has a magnitude $m_{\text{vega}} = +0.03$, is equal to $2.19 \times 10^{-8}\,\text{W/m}^2$.

Solution:
Let a_p be the cross-sectional area of the primary mirror

$$a_\text{p} = \pi \times r^2$$
$$= \pi \left(\frac{0.1524}{2}\right)^2$$
$$a_\text{p} = 0.0182\,\text{m}^2$$

Flux of Sirius at Earth:

$$m_{\text{sirius}} - m_{\text{vega}} = -2.5 \log \frac{F_{\text{sirius}}}{F_{\text{vega}}}$$

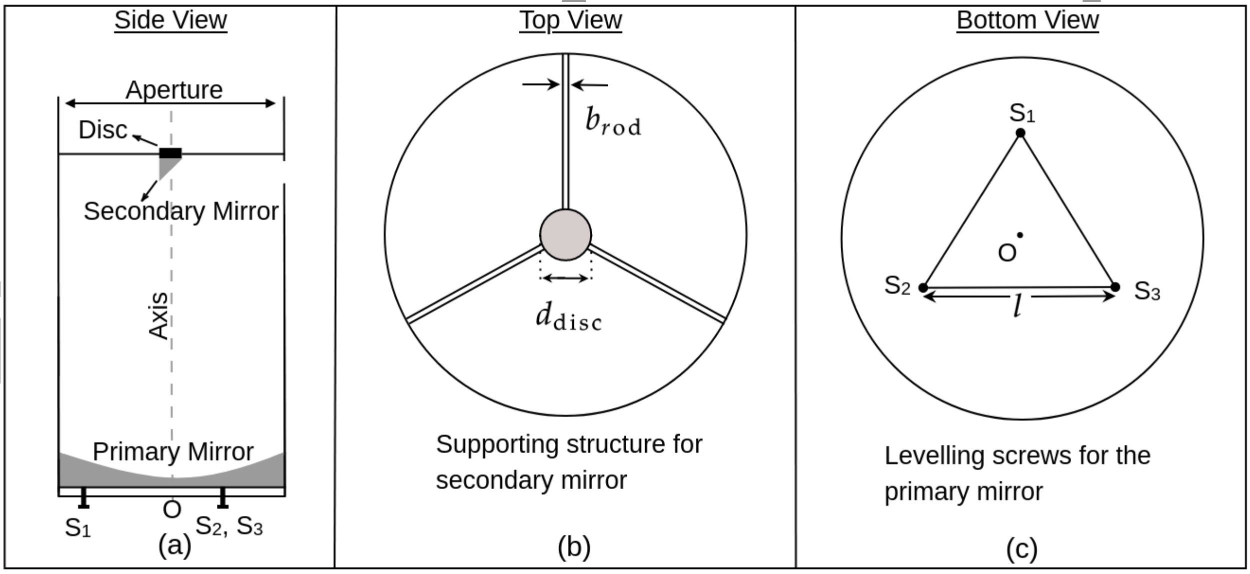
$$-1.46 - 0.03 = -2.5 \log \frac{F_{\text{sirius}}}{2.19 \times 10^{-8}}$$

$$\boxed{F_{\text{sirius}} = 8.64 \times 10^{-8}\,\text{W/m}^2}$$

Energy per unit time received from Sirius at the primary mirror of the telescope:

$$P_{\text{tel}} = F_{\text{sirius}} \times a_{\text{p}}$$
$$= 8.64 \times 10^{-8} \times 0.0182$$
$$\boxed{P_{\text{tel}} = 1.57 \times 10^{-9}\,\text{W}}$$

Consider a Newtonian reflecting telescope ((a) side, (b) top and (c) bottom views shown in the figure below) with a concave primary mirror of diameter 6 inches. A secondary mirror, mounted on a flat disc of diameter $d_{\text{disc}} = 3.7\,\text{cm}$, is held in place on the axis of the telescope tube by three thin rods of thickness $b_{\text{rod}} = 0.3\,\text{cm}$, as shown in the figure (b) below.



(b) (3 marks) Calculate the percentage reduction in flux at the primary mirror due to the supporting structure of the secondary mirror.

Solution:

$$\text{A}_{\text{disc}} = \pi r^2$$
$$= \pi \left(\frac{0.037}{2}\right)^2$$
$$= 1.075 \times 10^{-3}\,\text{m}^2$$

Length of the arms:

$$l_{\text{arm}} = 3 \times 2.54 - \frac{3.7}{2} = 5.77\,\text{cm}$$

Area blocked by the 3 arms

$$\begin{aligned} \text{A}_{\text{arms}} &= l_{\text{arm}} \times b_{\text{arm}} \times 3 \\ &= 0.0577 \times 0.003 \times 3 \\ &= 5.19 \times 10^{-4}\,\text{m}^2 \end{aligned}$$

Hence the total area obstructed by this assembly is:

$$\begin{aligned} a_{\text{obstructed}} &= \text{A}_{\text{disc}} + \text{A}_{\text{arms}} \\ &= 1.594 \times 10^{-3}\,\text{m}^2 \\ &= 0.0016\,\text{m}^2 \end{aligned}$$

Now, the flux will be reduced because of this assembly holding the secondary mirror. The percentage drop in the flux will be given by:

$$\begin{aligned} \text{Percentage drop} &= \frac{a_{\text{obstructed}}}{\text{a}_\text{p}} \\ &= \frac{0.0016}{0.0182} \end{aligned}$$

$$\boxed{\text{Percentage drop} = 8.79\%}$$

The optic axis of the primary mirror and the axis of the telescope tube are expected to be coincident. But sometimes, they may get misaligned due to mishandling or some other issues.

(c) (3 marks) Suppose that due to a misalignment, the angle between the two axes is $\theta = 5°$. Calculate the percentage change in the flux received at the primary due to this fault. For this part ignore the effects of the secondary mirror and its supporting structure. Assume that the diameter of the primary mirror is very close to that of the telescope tube.

Solution:

The misalignment tilts the mirror by 5°, hence the new projected area of the mirror in the telescope tube is

$$\begin{aligned} a_{\text{p-proj}} &= \pi \times a \times b \\ &= \pi a \times (a \cos\theta) \\ &= (\pi a^2) \cos\theta \\ a_{\text{p-proj}} &= 0.0182 \times \cos\theta \\ &= 0.0181\,\text{m}^2 \end{aligned}$$

Percentage change will be:

$$\text{Percentage change} = 1 - \cos 5^\circ$$
$$\boxed{\text{Percentage change} = 0.38\%}$$

The alignment of the axis of the primary mirror is done by three levelling screws, S_1, S_2 and S_3 at the bottom of the telescope. As seen in the bottom view (figure (c)), these screws form an equilateral triangle of side $l = 3$ inches. The optic axis of the primary mirror is perpendicular to the plane of the paper in figure (c) and passes through the centroid O of this triangle.

(d) (4 marks) Suppose that the 5° tilt of the axis of the mirror described above has happened in the plane containing the axis of the tube and the line OS_1 (see figure (c)). Therefore, it should be possible to correct the tilt and realign the axes by moving the screw S_1 alone, without disturbing S_2 or S_3. If the pitch of the screw S_1 is 1.15 mm, how many turns of this screw are needed to bring the mirror back in alignment?

Solution:
The distance of the third screw from the midpoint joining the two other screws is

$$h = 3 \times 2.54 \times \sin 60^\circ$$
$$= 6.60\,\text{cm}$$
$$\text{Now}, \sin 5^\circ = \frac{np}{h}$$
$$\text{Where, } n = \text{number of turns}$$
$$p = \text{pitch}$$
$$\therefore n = \frac{h}{p} \sin 5^\circ$$
$$\boxed{n = 5 \text{ turns}}$$

The maximum brightness of a source that can be observed with a telescope of a given aperture is limited by the tolerance of the human eye to bright light. The brightest object that can be observed safely with our telescope (primary mirror diameter of 6 inches) is of magnitude −10.50. To observe any source brighter than this limit the amount of light that reaches the primary mirror must be restricted by reducing the effective aperture of the telescope. This is accomplished by covering the top of the telescope with a lid that has a smaller hole (aperture) of appropriate size.

(e) (4 marks) Calculate the size of this smaller circular aperture on the lid so that a supernova of magnitude −12.525 can be safely observed. Once again, ignore the secondary mirror and its supporting structure.

Solution:

Flux reduction ratio desired is

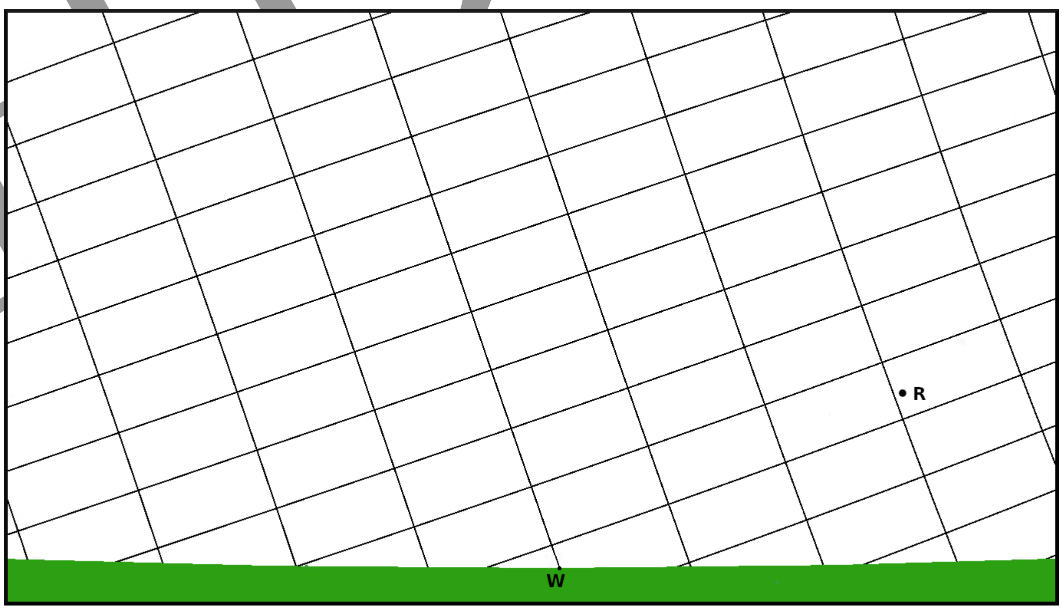
$$\mathrm{m}_4 - \mathrm{m}_{\mathrm{SN}} = -2.5 \log \frac{\mathrm{F}_4}{\mathrm{F}_{\mathrm{SN}}}$$
$$\frac{\mathrm{F}_{\mathrm{new}}}{\mathrm{F}_{\mathrm{SN}}} = 10^{\left(\frac{-2.025}{2.5}\right)}$$
$$= 0.155$$

We need a reduction in the number of photons received by this ratio, which can be accomplished by reducing the area of the telescope aperture. Therefore,

$$\left(\frac{r_{\mathrm{aperture}}}{r_{\mathrm{primary}}}\right)^2 = \frac{\mathrm{F}_{\mathrm{new}}}{\mathrm{F}_{\mathrm{SN}}}$$
$$= 0.155$$
$$\therefore r_{\mathrm{aperture}} = \sqrt{0.155} \times 7.62$$
$$\boxed{r_{\mathrm{aperture}} = 3.00\,\mathrm{cm}}$$

Therefore, the diameter of the smaller aperture would be 6.00 cm.

5. The image below shows a portion of the sky near the western horizon as observed from some place in the northern hemisphere. The horizon is shown as a green strip at the bottom of the image, and the letter 'W' marks the West cardinal point. The near-rectangular tilted grid on the sky is that of the RA and Dec coordinates. Each small rectangle corresponds to 10^{m} of RA and 5° of declination. A star, designated by the letter 'R', with its celestial coordinates (RA: $23^{\mathrm{h}}\ 04^{\mathrm{m}}$ & Dec: $+15°12'$) is shown in the image. Same image is provided in your answersheet to mark/write your answers. Answer the following questions with the information given above.



(a) (2 marks) Identify and mark the Celestial Equator on the image provided in the answersheet.

(b) (3 marks) Determine the approximate geographic latitude of the location.

(c) (4 marks) The table below contains RA and Dec of four stars. Mark these stars on the image in your answersheet.

Sr. No.	Star name	Right Ascension	Declination
A	70 Peg	$23^h\ 29^m$	$+12°45'$
B	β Psc	$23^h\ 4^m$	$+3°49'$
C	ι Cet	$0^h\ 19^m$	$-8°49'$
D	71 Aqr	$23^h\ 16^m$	$-9°5'$

(d) (2 marks) Identify the star(s) from the table that will NOT be observable to an observer located at the South Pole. Give reason in support of your answers.

(e) (3 marks) At the specified date and time, the coordinates of the (centre of the) Sun are RA: $23^h\ 54^m$ and Dec: $-0°36'$. On the image in your answersheet, accurately draw a disc of appropriate size representing the Sun at these coordinates.

(f) (2 marks) At the given location, if the star B in the table above sets at 18:01 hrs local time, determine the approximate local time at which another star, identified by coordinates RA: $23^h\ 17^m$ and Dec: $3°49'$, will set.

Solution:

(a) Refer image below.

(b) Here we find the angle made by the Celestial Equator with horizon (say θ):
Then the Geographic latitude of the place is : $90° - \theta \approx 19°$

> We accept angles between 19° to 20°

(c) Refer image below.

(d) Only stars with negative declination can be seen from the south pole. On the other hand stars having positive declination lie in the northern hemisphere of the Celestial sphere and hence will not be visible from the south pole.

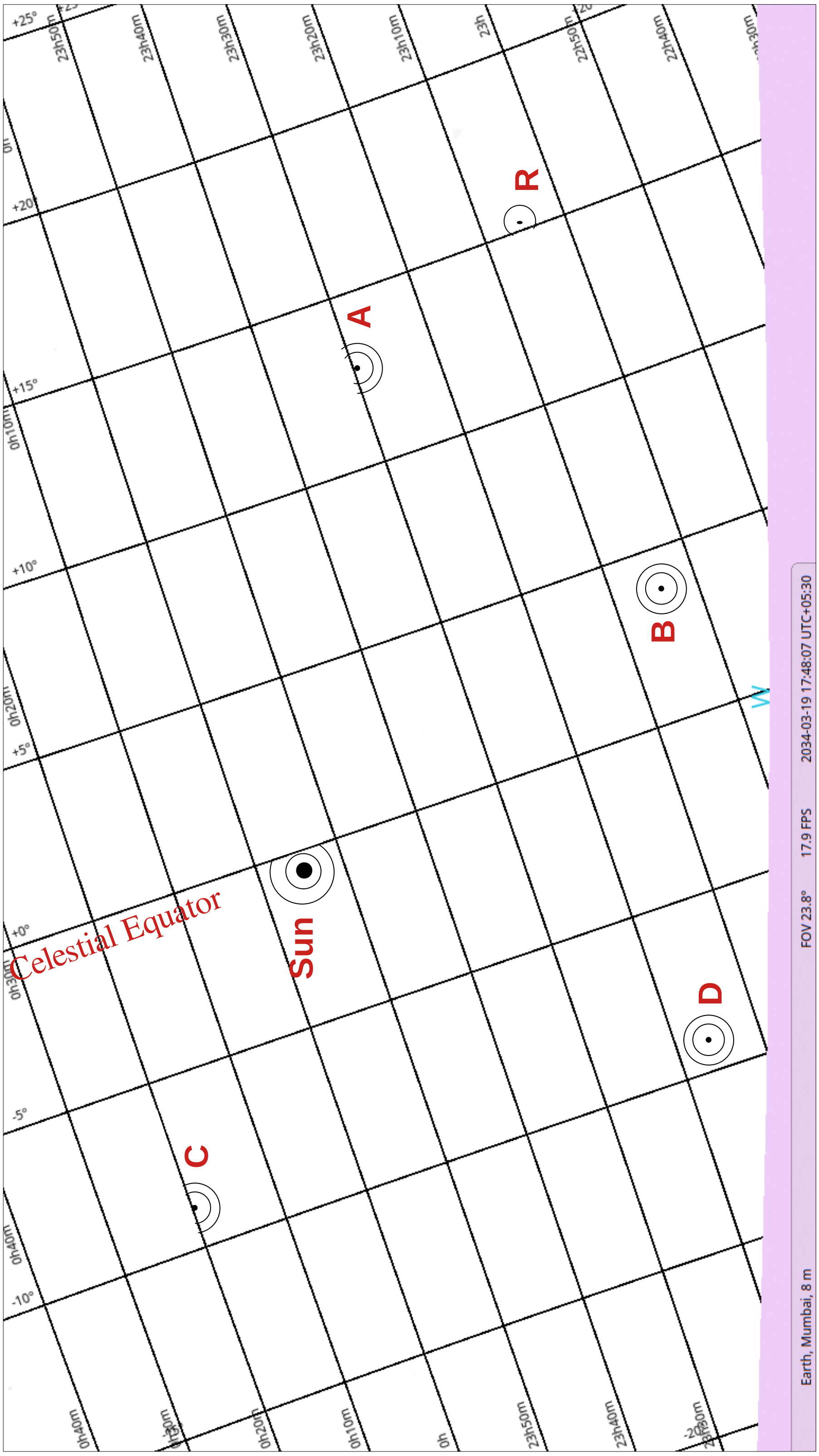
Sr. No.	Star name	Visibility	Reason
A	70 Peg	NO	Declination is positive.
B	β Psc	NO	Declination is positive.
C	ι Cet	YES	Declination is negative.
D	71 Aqr	YES	Declination is negative.

(e) Refer image below.

(f) In one hour, the sky moves roughly by one R.A. Hence, the approximate local time at which this another star will set is just the difference in R.A. of the two stars.

$$\text{Approx time} = 23^h\ 17^m - 23^h\ 4^m$$
$$= 13^m$$

Hence, the star will set at $18^h\ 14^m$.


Celestial Equator
Sun
A
B
C
D
R
W
+25°
+20°
+15°
+10°
+5°
+0°
-5°
-10°
23h50m
23h40m
23h30m
23h20m
23h10m
23h
22h50m
22h40m
0h
0h10m
0h20m
0h30m
0h40m
FOV 23.8°
17.9 FPS
2034-03-19 17:48:07 UTC+05:30
Earth, Mumbai, 8 m

Notes: Celestial Coordinate System

The *Celestial Sphere* is a representation of the sky as a huge imaginary sphere, with its centre at the centre of the Earth, on which all the celestial objects can be seen. The *Celestial Equator* is the circle marking the intersection of the Earth's equatorial plane with the celestial sphere. Thus, for an observer standing on the Earth's equator, the *Celestial Equator* will be a circle passing through the cardinal points East and West, and the Zenith (overhead point). The North and South *Celestial Poles* are the intersection points of the rotational axis of the Earth with the *Celestial Sphere*.

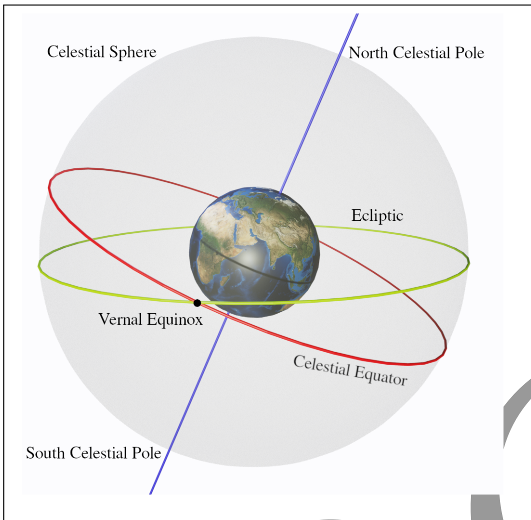
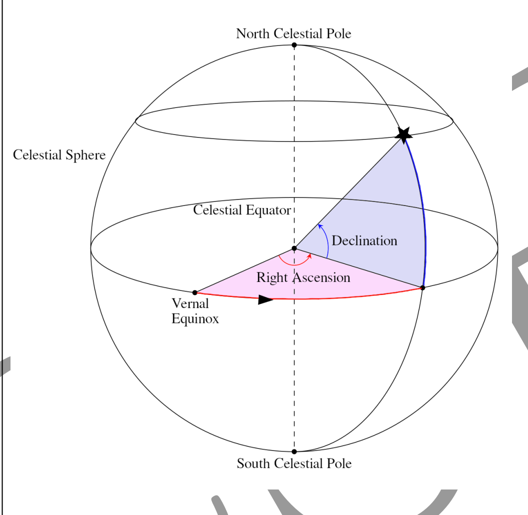


Diagram depicting the celestial sphere with Earth at its center.



Representation of a star's celestial coordinates (*RA* and *Dec*) on the celestial sphere.

With the equator and the poles thus defined, we can now define two celestial coordinates to describe the position of any celestial object on the *Celestial Sphere*, namely *Right Ascension (RA)* and *Declination (Dec)*. These celestial coordinates are analogous to the longitude and latitude on the Earth, respectively.
Right Ascension (RA) is the celestial analogue of the terrestrial longitude. The zero of RA passes through the Vernal Equinox, which marks the point where the Sun crosses the *Celestial Equator* into the northern part of the sky around 21 March every year. The corresponding terrestrial equivalent is the prime meridian, *i.e.,* 0° longitude, which passes through Greenwich, UK.
As we trace celestial equator in our sky from west to east, the *RA* value keeps increasing. Although *RA* can be expressed in degrees (like longitude), it is customary to express it in hours, minutes, and seconds of time — from $00^{\mathrm{h}}\,00^{\mathrm{m}}\,00^{\mathrm{s}}$ to $24^{\mathrm{h}}\,00^{\mathrm{m}}\,00^{\mathrm{s}}$ — thus "fixing" the *RA* grid to the sky, as the sky appears to rotate due to the diurnal rotation of the Earth around its axis. The entire celestial sphere completes a full 360° rotation in approximately 24 hours, equating to about 15° per hour. Therefore, in one hour, the sky rotates by about one hour of *RA*. (Conversion: $1^{\mathrm{h}} = 60^{\mathrm{m}} = 3600^{\mathrm{s}}$).
Declination (Dec) functions as the celestial counterpart of the terrestrial latitude, measured in degrees, arcminutes and arcseconds. Similar to latitude, positive and negative values indicate positions north and south of the *Celestial Equator*, respectively. The *Celestial Equator* has a declination of 0°, while the North and the South celestial poles have declinations of +90° and −90°, respectively. (Conversion: $1° = 60' = 3600''$).
Using these "equatorial coordinates", we can define the position of any object in the sky in much the same way as we use longitude and latitude to describe the location of any place on the Earth.