

Indian National Astronomy Olympiad (INAO) – 2023

Question Paper

Roll Number: - -

Duration: **Three Hours**

Date: 28 January 2023

Maximum Marks: 100

Please Note:

- Before starting, please ensure that you have received a copy of the question paper containing total 5 pages.
- Please write your roll number in the space provided above.
- There are total 6 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator.** You can take this question paper back with you.
- Please take note of following details about Orientation-Cum-Selection Camp (OCSC) in Astronomy:
 - Tentative Dates: 1 to 18 May 2023.
 - This camp will be held at HBCSE, Mumbai.
 - Attending the camp for the entire duration is mandatory for all participants.

Useful Constants

Mass of the Sun	$M_{\odot} \approx 1.989 \times 10^{30} \text{ kg}$
Mass of the Earth	$M_{\oplus} \approx 5.972 \times 10^{24} \text{ kg}$
Mass of the Moon	$M_{\zeta} \approx 7.347 \times 10^{22} \text{ kg}$
Radius of the Sun	$R_{\odot} \approx 6.955 \times 10^8 \text{ m}$
Radius of the Earth	$R_{\oplus} \approx 6.371 \times 10^6 \text{ m}$
Radius of the Moon	$R_{\zeta} \approx 1.737 \times 10^6 \text{ m}$
Speed of Light	$c \approx 2.998 \times 10^8 \text{ m/s}$
Astronomical Unit	$a_{\oplus} \approx 1.496 \times 10^{11} \text{ m}$
Solar Constant (at Earth)	$S \approx 1366 \text{ W/m}^2$
Gravitational Constant	$G \approx 6.674 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
Radius of Martian orbit	$d_{\text{Mars}} \approx 1.524 \text{ au}$

1. For the spacecrafts travelling from one planet to another, the travel path can be designed in multiple ways. The most energy efficient of these methods is called ‘Hohmann Transfer Orbit’. In this method, the spacecraft travels in an elliptical orbit with the Sun at the principal focus. Thus, its motion is governed by Kepler’s Laws and hardly any fuel is spent for this journey. Step-wise process of Hohmann orbit is as follows:

- Step 1: The spacecraft is in the close proximity of Earth.
- Step 2: The spacecraft engine is fired to allow it to escape the sphere of influence of the Earth and enter the region where the Sun’s gravity is the dominating force. The point in Earth’s orbit at which engine is fired will also be a point in the new orbit of the spacecraft around the Sun.
- Step 3: The spacecraft travels along an elliptical orbit around the Sun and reaches the orbit of Mars.
- Step 4: The timing of this journey is such that Mars is exactly at the common point of Martian orbit and the elliptical orbit of the spacecraft at the instance when the spacecraft reaches this point.
- Step 5: The spacecraft engine is fired again so that the spacecraft is captured by the Mars’ gravity and it remains in close proximity of Mars.

Points to note:

- A. The point where the engine is first fired (Step 2) will be spacecraft’s shortest distance from the Sun.
- B. The point where the spacecraft is captured by Mars (Step 5) will be the spacecraft’s farthest distance from the Sun.
- C. If Step 5 is not executed, the spacecraft will complete its orbit around the Sun and reach back the point in space where Step 2 was executed.
- D. Let us assume that both Earth and Mars are in circular, co-planer orbits around the Sun and the Hohmann transfer orbit is also in the same plane.
- E. Assume that all the three bodies (Earth, Mars and the spacecraft) are point masses and moving in anti-clockwise direction (as viewed from the top) in their respective orbits.
- F. The time durations for which engine is fired (Step 2 as well as Step 5) is very small. Hence, the thrust is almost instantaneous.

Based on this, answer the following questions:

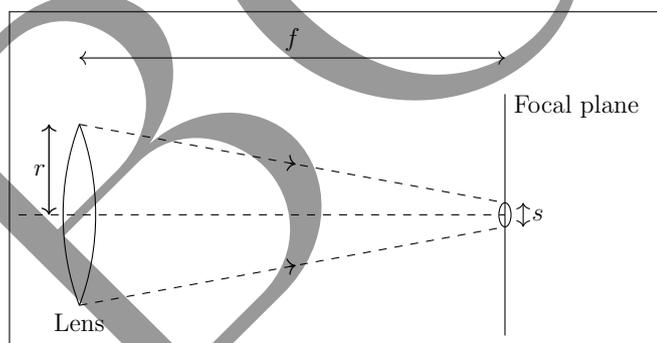
- (a) (1.5 marks) Find the orbital period of Mars in years.
- (b) (1.5 marks) Find the length of semi-major axis of this Hohmann transfer orbit (in au).
- (c) (1.5 marks) Find time spent by the spacecraft between Step 2 and Step 5.
- (d) (5 marks) Draw a diagram showing the Hohmann transfer orbit between Earth and Mars. Show positions of Earth and Mars at the instance of Step 2 (as E_1 and M_1) and at the instance of Step 5 (E_2 and M_2). Keep position E_1 directly below the Sun. Calculate angles subtended with respect to Sun - E_1 line for the other three points (M_1 , E_2 and M_2).
- (e) (2.5 marks) Calculate orbital velocity of Earth and Mars around Sun.

- (f) (8 marks) Determine if the following statements are True or False. Explain your reason in one line each.
- i. The semi-minor axis of the Hohmann orbit is 1 au.
 - ii. The semi-major axis of the Hohmann orbit is 1.52 au.
 - iii. The thrust was applied in Step 2 to increase the speed of the spacecraft.
 - iv. The thrust was applied in Step 5 to increase the speed of the spacecraft.
 - v. If Step 5 is missed, we can re-capture the spacecraft in an orbit around the Earth when it again reaches its departure point in space. (see point C above).
 - vi. The Hohmann transfer orbit only touches (and does not intersect) the Earth's orbit.
 - vii. The kinetic energy of the spacecraft in Hohmann orbit is lowest just before Step 5.
 - viii. The spacecraft would be approaching Mars from behind in its orbit.

2. (10 marks) A telescope with objective lens of focal length $f = 200$ cm is used to image a binary star system using a CCD kept at its focal plane and the image forming symmetrically at the centre (on the optic axis). In the figure, s is the image size of a point object. The image has a finite size due to diffraction. Two nearby objects are considered to be resolved when their image disks are just touching each other externally.

The radius of the objective lens is $r = 5$ cm. The limiting resolution of the lens is given to be $1''$ and the two stars are separated by an angle of $3''$. Determine the range dx allowed for the CCD to be displaced from the focal plane towards the lens such that individual stars can still be separately resolved.

(Note: This is related to the notion of “depth of field” of a camera lens.)



3. A cubical box, with edges of length s , is in a circular orbit of radius d around a star. The time period of revolution of the box is P . The star can be approximated as a perfect blackbody with radius R_0 ($s, R_0 \ll d$) and temperature T_0 .

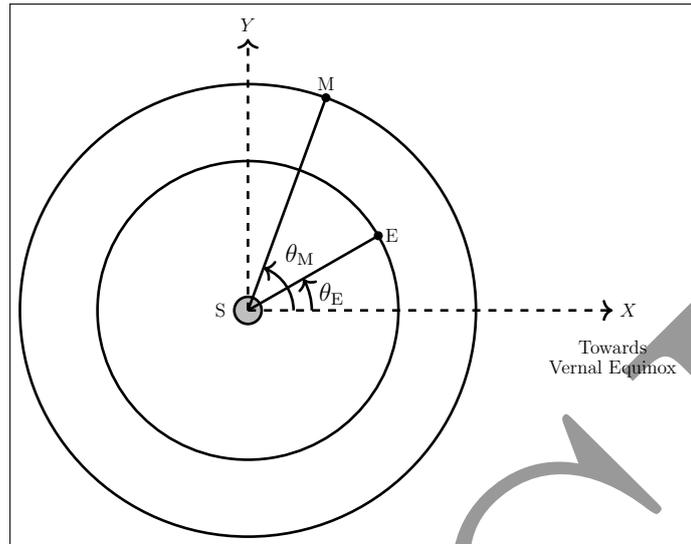
In all parts of this question, at time $t = 0$, normal vector to one face of the box is exactly pointing towards the star and this face has an albedo of A (where $A \in [0, 1]$). The top face of the box is parallel to the orbital plane of the box.

The box is in thermal equilibrium at each instance $t \geq 0$, i.e. its emissivity is exactly equal to its absorptivity. The temperature T in all parts of the box is the same at each instance.

Note: Albedo is the fraction of incident light reflected off the surface of an object.

- (a) (7 marks) If the same face of the box is locked in towards the star at all times, find its equilibrium temperature T .

- (b) (6 marks) Let all 6 faces of the box have albedo A . If the box is revolving around the star, but not rotating about itself, derive the expression for the equilibrium temperature T as a function of time t .
- (c) (3 marks) Now consider the case, where the face of the cube which is towards the star at $t = 0$ and the face opposite to it have albedo A and all other faces of the cube have albedo of $1 - A$. Observe the expressions obtained in part b and write the final expression for temperature in this case.
4. Indian Space Research Organisation (ISRO) decided to send a spaceship to the planet Jupiter. The spaceship needs to pass through the asteroid belt between Mars and Jupiter which lies between 2.0 au and 3.0 au. The asteroid belt has a thickness (total height perpendicular to the plane of the solar system) of about 0.10 au and there are about 10^{14} asteroids of 1.0 m radius (assumed to be spherical) in it. The spaceship is spherical and is 3.0 m in radius.
- (a) (2 marks) Calculate the cross-section area within which the centre of an asteroid needs to come from the centre of the spacecraft to collide with it.
- (b) (8 marks) Assume that the asteroid belt is completely static and the spaceship is launched on a random day of the year to travel radially away from Sun, but within the plane of solar system. Calculate the probability (P_0) that the spaceship will escape the asteroid belt unharmed.
- (c) Qualitatively compare the probability in the each of the following cases with P_0 :
- (2 marks) The probability (P_A) when the spacecraft's speed through the asteroid belt is doubled.
 - (1 mark) The probability (P_B) when the spacecraft is not moving radially but is in an elliptical orbit with aphelion point at Jupiter (5 au). Here we consider probability only for a single crossing.
 - (2 marks) The probability (P_C) when the spacecraft is not moving radially but is in an elliptical orbit with aphelion point at the centre of the asteroid belt (2.5 au).
5. (22 marks) Assume that the Earth and Mars revolve around the Sun in coplanar circular orbits of radius r_E and r_M respectively. In the figure below, θ_E and θ_M are the angles made by the radius vectors of Earth and Mars, measured at the Sun with respect to the X-axis (which points in the direction of Vernal Equinox - a reference point in space). On 21 September, θ_E was 0° and θ_M was 42.3° .
- In the answersheet you are given a table to note down your observations. Calculate the values of θ_E and θ_M for the dates mentioned in the table and write them in the respective columns. The answersheet also has a particular section of the orbits of Mars and Earth. The dotted lines are markings for angles in the orbits at an interval of 1° . The central black line is when θ_E or θ_M will be 90° . You can also see the distant stellar background near one edge of the page.
- For the dates given in the table mark the projected positions of Mars on the stellar background. Also write the corresponding observation numbers near the marked projected positions. Now describe the motion of Mars as seen by an Earth based observer.



6. Saeed is an observer at North Pole during the month of December. On the grid given in your answersheet:
- (1 mark) Mark the directions.
 - (1 mark) Mark the pole star (Polaris).
 - (6 marks) Approximately, for how many days will the Moon be visible in one lunar month assuming clear skies? If you were doing an exact calculation, which secondary effects you will have to consider? List any three effects that may affect your answer by an hour or more.
 - (6 marks) In this month the full Moon was seen on winter solstice day in the Gemini constellation. Saeed noted the Moon's position every day at 7 pm during an entire lunar month. Mark the approximate positions of the Moon as seen by her. For each observation label the day number starting with day of first observation.
 - (3 marks) Aadarsh, who is also camping at the North pole with Saeed, noted the Moon's position for 24 hours period of the 6th day after full moon. Describe what he will observe.