Indian National Physics Olympiad Theory Problems and Solutions (2006 - 2009)

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Indian National Physics Olympiad Theory Problems and solutions (2006 - 2009) First Edition, 2009

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Published byHomi Bhabha Centre for Science EducationTata Institute of Fundamental ResearchV. N. Purav Marg, Mankhurd,Mumbai - 400088, India.

Printed at Print Vision, Mumbai

This book can be purchased from, In-Charge, Publication Section, Homi Bhabha Centre for Science Education Tata Institute of Fundamental Research V. N. Purav Marg, Mankhurd, Mumbai - 400088, India. Phone: 91-22-25580036, 25554712, 25567711 Fax: 91-22-25566803

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Foreword

The Physics Olympiad activity has been attracting an ever growing number of students and teachers since it was launched in 1998. A key component of this activity is the Indian National Physics Olympiad Examination, popularly known as INPhO, which is conducted by the Homi Bhabha Centre for Science Education around January end each year. To meet the demands of the students and teachers a volume detailing INPhO problems from 1998-2005 was brought out in 2008. The present volume which covers the period from 2006-2009 maybe viewed as a continuation of the same – but with a difference. This time we have included the **solutions** also, and, in multiple formats: Brief Solutions and Detailed Solutions. Some of the problems have been slightly modified or corrected in order to read better. The INPhO has been a four hour exam. However, we want the reader to work on this book at a leisurely pace. Credit (marks) has been mentioned in order to give an idea of the relative difficulty level of each problem.

We urge the readers to attempt the problems and then look at the two sets of Solutions: Brief and Detailed. In this connection we want to share with the readers two stories about seminal discoveries in theoretical physics. Both of them were "problem solving" exercises and fetched the discoverers the Nobel Prize.

On Sunday October 7 1900, Max Planck was confronted with an experimental black body radiation curve in the high wavelength regime. It was a partial curve and it deviated from Wien's theoretical prediction. Planck undertook an exercise in which he attempted to reconcile Wien's law at low wavelength with the deviant experimentally observed behaviour at high wavelength. By evening he was able to fit the two pieces of this puzzle and come up with an interpolation formula. This heralded the birth of quantum mechanics and fetched Planck the Noble Prize in 1918.

In 1970 Kenneth Wilson of Cornell University was asked to give a friendly seminar on a research paper published jointly by two Italian physicists Di Castro and Jona-Lasinio. In other words his colleagues asked him to explain the work, something that we might ask our friends to do, say over a cup of tea. The deadline for the seminar was approaching and in a desperate bid to arrive at the final conclusions of the paper, Wilson invented his "own way", different from any other. This "own way" was the beautiful "renormalization group" approach which won him the Nobel Prize in 1982. In a similar fashion we once again urge the reader to attempt the problems first and develop their "own way" and not to simply look at the solutions.

In addition the book is peppered with additional comments and minor derivations. This adds value to the collection. It includes personal descriptions by Planck and Wilson about their above - mentioned discoveries. And a delightful perspective by Isidor Issac Rabi, the discoverer of nuclear magnetic resonance, on how not to approach a physics problem! Along with the historical remarks this aesthetic exercise raises the book from the level of being merely a "problem book" on physics.

We also invite the readers to write to us pointing out errors and alternate solutions. Last, but certainly not the least, we would like to thank Ms. Sarita Yadav for discussions and excellent technical assistance.

Dated: July 1, 2009 **Prof. (Dr.) Vijay A. Singh** National Co-ordinator, Science Olympiads Homi Bhabha Centre for Science Education (TIFR)

Table of Constants

Acceleration due to gravity on	g	$9.80665 \text{ m} \cdot \text{s}^{-2}$
Earth	_	
Atmospheric pressure	P_{atm}	1.01325×10^5 Pa
Atomic mass unit	1 u	931.49403 $MeV \cdot c^{-2}$
Avogadro number	NA	$6.02214 \times 10^{23} \text{ mol}^{-1}$
Boltzman constant	k	$1.38065 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$
Distance between Sun and Earth	1 A.U.	$1.49600 \times 10^{11} \text{ m}$
Binding energy of hydrogen atom	-	13.6058 eV
Magnitude of electron charge	e	$1.60218 \times 10^{-19} \text{ C}$
Mass of the Earth	M_E	$5.97420 \times 10^{24} \text{ kg}$
Mass of the electron	m_e	$9.10938 \times 10^{-31} \text{ kg}$
Mass of the proton	m_p	$1.67262 \times 10^{-27} \text{ kg}$
Mass of the Sun	M_{\odot}	$1.98892 \times 10^{30} \text{ kg}$
Permeability of free space	μ_0	$1.2566 \times 10^{-6} \text{ H} \cdot \text{m}^{-1}$
Permittivity of free space	ϵ_0	$8.85420 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}$
Planck's constant	h	$6.62607 \times 10^{-34} \text{ J} \cdot \text{s}$
Radius of the Earth	R_E	$6.37814 \times 10^{6} \text{ m}$
Radius of the Sun	R_{\odot}	$6.95500 \times 10^8 \text{ m}$
Speed of Sound in air	c_s	$340.29 \text{ m} \cdot \text{s}^{-1}$
(at room temperature)		
Speed of light in vacuum	c	$2.99793 \times 10^8 \text{ m} \cdot \text{s}^{-1}$
Stefan-Boltzmann constant	σ	$5.67040 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$
Surface Tension of water at 20° C	-	$7.286 \times 10^{-2} \text{ N} \cdot \text{m}^{-1}$
Universal constant of Gravitation	G	$6.67428 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$
Universal gas constant	R	$8.31447 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$
Wien's constant	_	$2.89777 \times 10^{-3} \text{ m} \cdot \text{K}$

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Chapter I

Problems

1 INPhO-2006

Indian National Physics Olympiad - 2006

т

INPhO-2006

Jan. 29, 2006 Maximum Marks: **90**

 m_2



e diagram shown (Fig. (1)), $m_1 = 1$ kg, $m_2 = 1$ kg and c

- 1. In the diagram shown (Fig. (1)), $m_1 = 1$ kg, $m_2 = 1$ kg and coefficient of friction, both static and dynamic, between m_1 and plane is $\mu = 0.6$. The two masses are connected by a light inextensible string passing over a light frictionless pulley. Take $g = 10 \text{ m} \cdot \text{s}^{-2}$.
 - (a) Find the acceleration of the system.
 - (b) Find the force of friction and the magnitude of the tension in the string.

[10]

- 2. A block of uniform mass M is at rest on a table. A disk of mass 2M, radius R and of the same height as the block, which is initially spinning about its axis with angular speed ω_0 , is placed on the table such that it touches the block (see Fig. (2)). The block disk system starts moving such that they are in contact throughout the motion. Coefficient of friction, both kinetic and static, between the table and block and between the table and disk is μ . Friction between disk and the block may be ignored.
 - (a) Obtain an expression for the initial acceleration of the block disk system.



Figure 2: Problem 2

- (b) Obtain an expression for the time t^* at which pure rolling (i.e. rolling without slipping) starts.
- (c) Obtain an expression for the total time t_{tot} in which the block comes to the rest. Assume that pure rolling persists for $t > t^*$.

[14]

3. An ideal gas goes through a reversible cycle which consists of two isobaric and two adiabatic processes as shown in the P - V diagram (Fig. (3)).



- (a) Obtain an expression for the efficiency of the cycle in terms of the temperatures $\{T_a, T_b, T_c, T_d\}$.
- (b) Obtain an expression for the efficiency of the cycle in terms of the pressures $\{P_1, P_2\}$ and γ . Here γ is the ratio of the specific heat at constant pressure and specific heat at constant volume.
- (c) Draw the equivalent V T diagram for this cycle. [Note: V along y-axis and T along x-axis.]
- (d) State the expression for the corresponding Carnot cycle working with the same gas and between the highest and lowest temperatures defined by the above cycle. Which of these two cycles has the higher efficiency?

[12]

4. A thin plano-convex lens of radius R = 10 cm, refractive index $\mu_2 = 1.5$ has its curved surface in liquid of refractive index $\mu_3 = 1.2$ and the plane surface exposed to air of refractive index $\mu_1 = 1.0$. A self luminous particle oscillating simple harmonically with small amplitude $\sqrt{2}$ cm is placed on the axis of the lens as shown in Fig. (4). Determine the orientation, amplitude and phase



difference of the oscillating final image with respect to the object.

[8]

5. A thin circular disk of radius R is uniformly charged with charge σ ($\sigma > 0$) per unit area. The disk rotates about its axis OX with a uniform angular speed ω (see Fig. (5)). A small magnetic dipole of moment $\vec{\mu}$ is located at P(a, 0, 0) on the axis of the disk (a > 0).



- (a) Obtain the expression for the magnetic moment of the disk?
- (b) Obtain the expression for the magnetic field \vec{B} due to the rotating disk at P?
- (c) Obtain the approximate expression for \vec{B} when a >> R.
- (d) Obtain the force on the dipole placed at P given that a >> R.

[Note: You can use the formula for \vec{B} on the axis of a circular current, namely

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2i\pi r^2}{\left(r^2 + x^2\right)^{3/2}} \qquad]$$
[14]

6. A 1.00 kW cylindrical (monochromatic) laser light beam of radius δ is used to levitate a solid aluminium sphere of radius R by focusing it on the sphere from below (see Fig. (6)). The laser light is reflected by the aluminium sphere without any absorption.



- (a) Take the momentum of each photon in the light beam to be p. Express the force exerted on the aluminium sphere by the beam in terms of p, δ , R, and n where n is the number of photons per unit area per unit time.
- (b) Now consider the special case $\delta \ll R$. Calculate the mass of the sphere, assuming that it floats freely on the light beam?

[Hint: Part (b) can be done independently of Part (a)] [10]



Figure 7: Demonstrations of laser levitation

- 7. An electron in the Li^{++} ion makes a transition from n = 4 to n = 3 state.
 - (a) Find the wavelength of emitted photon in this process. To what region of the electro-magnetic spectrum does this wavelength belong?

(b) This photon impinges on a photoelectric sensitive metal having work function 3.20 eV. Calculate the maximum kinetic energy and the corresponding de Broglie wavelength of emitted photo-electron.

(Ionization energy of hydrogen atom = 13.60 eV)

[8]

[14]

8. Lyttleton - Bondi Model for the Expansion of the Universe *

In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained on the basis of Newtonian mechanics if matter carried a net electric charge. Imagine a spherical volume of astronomical size and radius R containing un-ionized atomic hydrogen gas of uniform density $\eta (= 10^{-26} \text{kg} \cdot \text{m}^{-3})$, and assume that the proton charge $e_p = -(1 + y)e$, where e is the electron charge.

- (a) Obtain the value of y for which the electrostatic repulsion becomes larger than the gravitational attraction and the gas expands.
- (b) Obtain an expression for the force of repulsion on an atom which is at a distance R from the centre of the spherical volume. Hence show that the radial velocity is proportional to R. Let us label the proportionality constant as H. Assume that the density is maintained constant by the continuous creation of matter in space. Assume also that the value of y is larger than the equilibrium value calculated in part (a) above and hence ignore gravity.
- (c) Calculate the numerical value of H. Take the value of y to be one order of magnitude larger than the equilibrium value calculated in part (a) above
- (d) Given that at time t = 0, the volume of the Universe was V_0 , obtain an expression for the volume expansion of the Universe.
- (e) Why do you think the Lyttleton Bondi model has been largely discarded by the scientific community?

*Ref. R.A.Lyttleton and H.Bondi, Proceedings of Royal Society of London, Volume A 252, page 313 - 333, (1959)



Raymond Arthur Lyttleton (7 May 1911-16 May 1995) : English mathematician and theoretical astronomer who researched stellar evolution and composition. In 1939, with Fred Hoyle, he demonstrated the large scale existence of interstellar hydrogen, refuting the existing belief that space was devoid of interstellar gas. Together, in the early 1940's, they applied nuclear physics to explain how energy is generated by stars. In his own monograph (1953) Lyttleton described stability of rotating liquid masses, which he extended later to explain that

the Earth had a liquid core resulting from a phase change associated with a combination of intense pressure and temperature. With Hermann Bondi, in 1959, he proposed the electrostatic theory of the expanding universe. He authored various astronomy books. One of them "Mysteries of the Solar System", was co-authored with Edwin Land and was quite popular.



Sir Hermann Bondi (1 Nov.1919-10 Sept.2005) : Austrian-born British mathematician and cosmologist who, with Fred Hoyle and Thomas Gold, formulated the steady-state theory of the universe (1948). Their theory addressed a crucial problem: "How do the stars continually recede without disappearing altogether?" Their explanation was that the universe is ever-expanding, without a beginning and without an end. Further, they said,

since the universe must be expanding, new matter must be continually created in order to keep the density constant, by the interchange of matter and energy. The theory was eclipsed in 1965, when Arno Penzias and Robert Wilson discovered a radiation background in microwaves giving convincing support to the "big bang" theory of creation which is now accepted.

"Sometimes I am a little unkind to all my many friends in education ... by saying that from the time it learns to talk every child makes a dreadful nuisance of itself by asking 'Why?'. To stop this nuisance society has invented a marvellous system called education which, for the majority of people, brings to an end their desire to ask that question. The few failures of this system are known as scientists."



2 INPhO-2007

Indian National Physics Olympiad - 2007

INPhO-2007

Jan. 28, 2007 Maximum Marks: **80**

1. The polar coordinates of a particle of mass m moving in a trajectory under the influence of a force \vec{F} are given by: r = at and $\theta = \omega t$, where a and ω are constants. Note acceleration in polar coordinates is

$$a_r = \ddot{r} - r\dot{\theta}^2, \ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

- (a) State the momentum vector \vec{p} and the force vector \vec{F} .
- (b) Evaluate the work done $\Delta W = \int \vec{F} \cdot d\vec{r}$ explicitly if the initial radial distance of the particle is negligible and the final distance is r.
- (c) Sketch the trajectory.

[6]

2. A small spherical ball undergoes an elastic collision with a rough horizontal surface. Before the collision, it is moving at an angle θ to the horizontal (see Fig. (8)). You may assume that the frictional force obeys the law $f = \mu N$ during the contact period, where N is the normal reaction on the ball and μ is the coefficient of friction.



- (a) Obtain $\theta_m(\mu)$ so that the subsequent horizontal range of the ball after leaving the horizontal surface is maximized.
- (b) State the allowed range of θ_m .

[10]

3. A cylindrical block of length 0.4 m and uniform area of cross section 0.04 m² is placed in concentric contact with a metal disc of mass 0.4 kg and of the same cross section (see Fig. (9)). The left face (A) of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is $\theta_i = 280.0 \text{ K}$. If the thermal conductivity of the material of the cylinder is 10 W·m⁻¹·K⁻¹ and the specific heat of the material of the disc is

$$C = C_0[1 + \alpha(\theta - \theta_i)]$$

where $C_0 = 600.0 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ and $\alpha = 0.010 \text{ K}^{-1}$, then:



Figure 9: Problem 3

- (a) How long will it take for the temperature of the disc to increase to 340 K? Assume that there is no heat loss from the disc.
- (b) Repeat the exercise of part (3a) if the specific heat of the disc was $C = C_0$, i.e. temperature independent.
- (c) Which process, (3a) or (3b) takes longer time? Why?

Assume that no heat is lost by radiation or convection and that the process of heat transfer is solely conduction. [10]

4. A transparent sphere of radius R and refractive index n is at rest on a horizontal surface. A ray of light is incident parallel to the vertical diameter and at a distance d from it.



Figure 10: Problem 4

- (a) Obtain an expression for d in terms of refractive index n and radius R such that the ray intersects the diameter at the point of emergence (see Fig. (10)).
- (b) What is the allowed range of n for the above possibility to occur?

[8]

- 5. (a) Find the electric field due to an infinite line of charge with linear charge density λ at a distance r from the line.
 - (b) Using a point at perpendicular distance a from the line charge (i.e. r = a) as a reference, find the potential at a distance r from the line.

(c) Now two line charges, with densities λ and $-\lambda$ are kept distance 2d apart as shown in Fig. (11). Consider a plane perpendicular to the line charges (e.g. the plane of this paper). Obtain explicit expression for the equipotential lines in this plane.





- (d) Make a clear plot of these equipotential lines and comment briefly on them. State the shape of the equipotential surfaces.
- (e) Now these line charges start moving parallel to each other with speed v. Obtain the speed at which the magnitudes of electric and magnetic forces are equal to each other.

[10]

6. An equilateral triangle of side S carrying a current I_1 is placed with its base at a distance *a* from an infinite straight wire carrying a current I_2 parallel to the base (see Fig. (12)).



- (a) Find the force on the triangle.
- (b) Sketch the magnitude of this force as a function of S/a.

[6]

7. A narrow beam of monochromatic light from source S of wave length 6000.0 Å moves along the positive x-axis and is incident on mirror M. The area vector of M is $0.04(-\hat{\imath} + \hat{\jmath})$ m². The mirror has reflectivity unity, in other words the mirror is a perfect reflector. An electrically insulated metal surface of total area 0.04 m² is placed parallel to x-axis and above the mirror to receive the reflected beam (see Fig. (13)). The work function of the metal is 1.90 eV, its photoelectric efficiency is 10.0% and generated photoelectrons are immediately removed from the neighbourhood. The power of the source is 60.0 W. Assume the metal surface to be large and ignore edge effects. Find out:



Figure 13: Problem 7

- (a) The force exerted by the beam on the mirror.
- (b) The surface charge density on the metal surface after 10 seconds.
- (c) The energy density due to the electric field after 10 seconds.
- (d) The range of kinetic energy of emitted electrons.

[10]

8. Height of the Atmosphere

Consider a simplified model for the height to which the atmosphere extends above the earth's surface. In this model the atmosphere consists of the diatomic gases oxygen and nitrogen in the proportion 21:79 respectively. We assume that the atmosphere is an ideal gas and air processes are adiabatic.

- (a) Obtain an expression for the lapse rate Γ (change in temperature T with height z above the earth's surface) in terms of γ , R, g and m_a . Here γ is the ratio of specific heat at constant pressure to specific heat at constant volume; g, the acceleration due to gravity; R, the gas constant; and m_a , the relevant atomic mass.
- (b) What is the change in temperature when we ascend a height of one kilometer?
- (c) Consider the above model and express the pressure as a function of the height z, the lapse rate Γ and the constants $\{m_a, g, \text{ and } R\}$. You may assume that at z = 0, the surface temperature $T = T_0$ and pressure $p = p_0$.
- (d) According to this model what is the height to which the atmosphere extends? Take $T_0 = 300$ K and $p_0 = 1$ atm.

[10]

9. The Metal Detector

We consider a simple model of the metal detector with a coil (field coil) of radius R_f and concentric and coplanar smaller coil (called the pick-up coil) of radius R_p . The number of turns in the field and pick-up coils are N_f and N_p respectively. A sinusoidal current I(t) is passed through field coil.

- (a) State the magnetic field B at the centre of the set-up due to I(t).
- (b) We approximate the magnetic field throughout the interior of the smaller coil by the magnetic field calculated in part (9a). Obtain an expression for the induced emf in the pick-up coil.[Note that this approximation underestimates the flux by about the 10%.]
- (c) Given the following values: f = 5000 Hzmaximum current $I_0 = 0.5 \text{ A}$ maximum induced emf $E_0 = 0.25 \text{ V}$ $R_p = 0.025 \text{ m}$ $R_f = 0.05 \text{ m}$. Calculate the product $N_p N_f$.
- (d) What is the mutual inductance on the field coil due to the pick up coil?
- (e) The optimization problem is to use the least amount of wire with the given quantities in part (9c) being kept fixed. Under these constraints determine the allowed ranges of N_p and N_f individually.
- (f) Qualitatively describe what happens to the induced emf when you place small disks of the following material at the centre of the pick-up coil:
 - i. Iron
 - ii. Wood
 - iii. Copper

[10]



Metal Detector: The operation of metal detectors is based upon the principle of electromagnetic induction. Metal detectors contain one or more inductor coils that are used to interact with metallic elements which are often hidden or invisible. A pulsing current is applied to the coil, which then induces a magnetic field. When the magnetic field of the coil moves across metal, such as a coin, the field induces electric currents (called eddy cur-

rents) in the coin. The eddy currents induce their own magnetic field which generates an opposite current in the coil, which in turn induces a signal indicating the presence of metal.



3 INPhO-2008

Indian National Physics Olympiad - 2008

INPhO-2008

Feb. 03, 2008 Maximum Marks: **80**

1. We define three quantities as follow:

$$A = m_e c^2, \ B = h/m_e c, \ C = e^2/2\epsilon_0 ch$$

where m_e is electron mass and other symbols have their usual meanings. For the hydrogen atom, express the radius of the n^{th} Bohr orbit r_n , the energy level E_n , and the Rydberg constant R in terms of any two of $\{A, B, C\}$.

[5]

2. Consider a ball which is projected horizontally with speed u from the edge of a cliff of height H as shown in the Fig. (14). There is air resistance proportional to the velocity in both x and y direction i.e. the motion in the x (y) direction has air resistance with the deceleration given by the $cv_x(cv_y)$ where c is the proportionality constant and $v_x(v_y)$ is the component of the instantaneous velocity in the x (y) direction. Take the downward direction to be negative. The acceleration due to gravity is g. Take the origin of the system to be at the bottom of the cliff as shown in Fig. (14).



Figure 14: Problem 2

- (a) Obtain expressions for x(t) and y(t).
- (b) Obtain the expression for the equation of trajectory.
- (c) Make a qualitative, comparative sketch of the trajectories with and without air resistance.
- (d) Given that height of cliff is H = 500 m and c = 0.05 s⁻¹, obtain the approximate time in which the ball reaches the ground. Take g = 10 m·s⁻².

[12]

3. Free Standing Tower

Consider a tower of constant density (ρ) and cross sectional area (A) (see Fig.

(15)) at the earth's equator. The tower has a counter weight at one end. It is free standing. In other words its weight is balanced by the outward centrifugal weight so that it exerts no force on the ground beneath it and tension in the tower is zero at both ends. Consider the earth to be an isolated heavenly body and ignore gravitational effects due to the other heavenly bodies such as moon. Further assume that there is no bending of the tower.



Figure 15: Problem 3

- (a) Draw the free body diagram of the small element of this tower at distance r from the centre of the earth.
- (b) Let T(r) be the tensile stress (tension per unit area) in the tower. Use Newton's equations to write down the equation for dT(r)/dr in terms of G, ρ, geostationary height R_g from the earth's centre and earth's mass M.
- (c) Taking the boundary condition (T(R) = T(H) = 0), obtain the height of tower H in terms of R and R_g . Note that R is the radius of earth. Calculate the value of H.
- (d) The tensile stress in the tower changes as we move from r = R to r = H. Sketch this tensile stress T(r).
- (e) Steel has density of $\rho = 7.9 \times 10^3 \text{ kg} \cdot \text{m}^{-3}$. Its breaking tensile strength is 6.37 GPa. Calculate the maximum stress in the tower. State if a tower made of steel would be feasible.

Note: $M = 5.98 \times 10^{24}$ kg; R = 6370 km ; $R_g = 42~300$ km

[12]



Space elevator: The space elevator seems like an idea out of a science fiction movie. Put simply its a giant elevator from earth running up to a satellite in space. As crazy as it sounds, a lot of people believe it could work. The technology is based on nanotubes, and they believe that they could create a ribbon cable that could hold a tremendous amount of weight. A runner car will then go up and down on this cable.

4. Two identical walls, each of width $w \ (= 0.01 \text{ m})$, are separated by a distance $d \ (= 0.10 \text{ m})$ as shown in Fig. (16). Temperatures of the external face of the walls are fixed $(T_1 \text{ and } T_2, T_2 > T_1)$. Coefficient of thermal conductivity of wall is $k_w = 0.72 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$. We define

$$T_0 = \frac{T_1 + T_2}{2}, \quad \Delta = T_2 - T_1 \text{ and } \delta = T'' - T'$$
 (1)

where T' and T'' are the temperatures of the internal face of the walls 1 and 2 respectively. Then δ will depend on the type of heat transfer process in central region (of width d) between the walls i.e. on the conduction, radiation or convection heat transfer. Assume that the heat transfer is a steady state process.



Figure 16: Problem 4

- (a) Write down the expression for heat transfer flux q_w (W·m⁻²) inside the wall 1 in terms of k_w , T_1 , T', and w. Similarly also write the expression for wall 2.
- (b) Rewrite q_w in terms of Δ , δ , k_w , and w.

As mentioned above, in the central region between the walls, heat is transmitted by conduction, convection and radiation. Also due to the steady state process, the corresponding fluxes are equal to q_w . In what follows we will calculate the heat transfer fluxes between the walls due to these three processes each of these processes being considered separately.

Radiation process will take place without the presence of material medium in the central region between the walls. We assume that the central region between the walls is vacuum. Let ϵ be the emissivity of the walls and E_1 and E_2 be the total heat flux due to radiation from wall 1 to 2 and vice versa. Thus $E_1 = \epsilon \sigma T'^4 + (1 - \epsilon)E_2$ where σ is the Stefan-Boltzmann constant. Similarly one may write the equation for E_2 .

- (c) The net heat transfer is $q_r = E_2 E_1$. Write the expression for q_r in terms of ϵ , T'', and T'.
- (d) Rewrite q_r in terms of $\{k_w, \Delta, T_0, \sigma, \epsilon \text{ and } w\}$. [Hint: Eliminate δ using $\delta^2 \ll T_0^2$.]
- (e) Calculate q_r if $\epsilon = 0.9$.

In the following two parts we are considering only convection between the walls.

(f) Now we assume that central region is filled with air of coefficient of thermal conductivity k_a . In this condition, convected heat transfer between walls will take place. Equation for flux due to this process is given by

$$q_{cv} = \frac{N_u k_a}{d} (T'' - T')$$

where N_u is called the Nusselt number and for the given system $N_u = 6.4$. Due to the steady state nature of the process $q_w = q_{cv}$. Express q_{cv} in terms of $\{k_w, k_a, \Delta, w, d, \text{ and } N_u\}$.

- (g) Calculate the value of q_{cv} if $k_a = 0.026 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$.
- (h) Instead of air, the central region is now filled with sheathing material having coefficient of thermal conductivity k_s . Hence heat transfer will take place by conduction between walls. Express heat transfer flux q_{cd} in terms of $\{k_s, k_w, d, w, \text{ and } \Delta\}$. We assume that no radiation passes through sheathing material.
- (i) Taking $k_s = 0.05 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$, calculate the value of q_{cd} .
- (j) Considering all possible heat transfer process in the central region between the walls, which insulation (sheathing, air, or vacuum) is the most efficient?

[16]

- 5. Sunlight falls on the convex surface of the plano convex lens of aperture 0.080 m. The radius of curvature of the convex surface of the lens is 0.100 m. The refractive indices of the material of the lens for extreme red and violet colours of sunlight are 1.600 and 1.700 respectively. [Given that: Radius of the Sun = 6.96×10^8 m, Distance between Sun and Earth = 1.5×10^{11} m.]
 - (a) Calculate the positions of the observed image of the Sun with violet and red centre.
 - (b) Calculate the sizes of the observed image of the sun with violet and red centre.

[10]

6. Determination of The Speed of Light:

The speed of light maybe determined by an electrical circuit using low frequency ac fields only. Consider the arrangement shown in the Fig. (17). A sinusoidally varying voltage $V_0 \cos(2\pi ft)$ is applied to a parallel plate capacitor C_1 of radius a and separation s and also to the capacitor C_2 . The charge flowing into and out of C_2 constitutes the current in the two rings of radii band separation h. When the voltage is turned off the two sides (the capacitor C_1 on one side and the rings on the other) are exactly balanced. Ignore wire resistance, inductance and gravitational effects.



- (a) Obtain an expression for the time-averaged force between the plates of C_1 .
- (b) Obtain an expression for the time-averaged force between the rings. The magnetic force between the two rings maybe approximated by those due to long straight wires since b >> h.
- (c) Assume that C_2 and the various distances are so adjusted that the timeaveraged downward force on the upper plate of C_1 is exactly balanced by the time-averaged downward force on the upper ring. Under these conditions obtain an expression for the speed of light.

(d) Numerically estimate the speed of light given that: a = 0.10 m, s = 0.005 m, b = 0.50 m, h = 0.02 m, f = 60.0 Hz, $C_1 = 1.00$ nF (nano-farad) and $C_2 = 632 \ \mu$ F (micro-farad).

[Hint: Not all the given quantities are required to obtain the estimate.]

[12]

- 7. An N turn metallic ring of radius a, resistance R, and inductance L is held fixed with its axis along a spatially uniform magnetic field \vec{B} whose magnitude is given by $B_0 \sin(\omega t)$.
 - (a) Set up the emf equation for the current i in the ring.
 - (b) Assuming that in the steady state i oscillates with the same frequency ω as the magnetic field, obtain the expression for i.
 - (c) Obtain the force per unit length. Further obtain its oscillatory part and the time-averaged compressional part.
 - (d) Calculate the time-averaged compressional force per unit length given that $B_0 = 1.00$ tesla, N = 10, a = 10.0 cm, $\omega = 1000.0$ rad·s⁻¹, R = 10.0 Ω , L = 100.0 mH.
 - (e) Answer the following two questions without providing rigorous justification:
 - i. For $\omega/2\pi = 60$ Hz, the ring emits a humming sound. What is the frequency of this sound?
 - ii. A capacitor is included in the circuit. How does this affect the force on the ring?

[13]

4 INPhO-2009

Indian National Physics Olympiad - 2009

INPhO-2009

Feb. 01, 2009 Maximum Marks: **70**

Note: Questions 1-38 is a set of multiple choice questions. Only one of the given choices is the best choice. Select this most appropriate choice.

- 1. A block of weight 200 N is at rest on a rough inclined plane of inclination angle $\theta = 30^{\circ}$. The inclined plane is at rest in the earth's inertial frame. Then the magnitude of the force the plane exerts on the block is
 - (a) $100\sqrt{3}$ N.
 - (b) 100 N
 - (c) 200 N
 - (d) zero.
- 2. A spatially uniform magnetic field \vec{B} exists in the circular region S and this field is decreasing in magnitude with time at a constant rate (see Fig. (18)).



Figure 18: Problem 2

The wooden ring C_1 and the conducting ring C_2 are concentric with the magnetic field. The magnetic field is perpendicular to the plane of the figure. Then

- (a) there is no induced electric field in C_1 .
- (b) there is an induced electric field in C_1 and its magnitude is greater than the magnitude of the induced electric field in C_2 .
- (c) there is an induced electric field in C_2 and its magnitude is greater than the induced electric field in C_1 .
- (d) there is no induced electric field in C_2 .
- 3. During negative β decay, an anti-neutrino is also emitted along with the ejected electron. Then
 - (a) only linear momentum will be conserved.
 - (b) total linear momentum and total angular momentum but not total energy will be conserved.
 - (c) total liner momentum and total energy but not total angular momentum will be conserved.

- (d) total linear momentum, total angular momentum and total energy will be conserved.
- 4. Five identical balls each of mass m and radius r are strung like beads at random and at rest along a smooth, rigid horizontal thin rod of length L, mounted between immovable supports (see Fig. (19)).



Assume 10r < L and that the collision between balls or between balls and supports are elastic. If one ball is struck horizontally so as to acquire a speed v, the magnitude of the average force felt by the support is

(a)
$$\frac{5mv^2}{L-5r}$$

(b)
$$\frac{mv^2}{L-10r}$$

(c)
$$\frac{5mv^2}{L-10r}$$

(d)
$$\frac{mv^2}{L-5r}$$

5. In Young's double slit experiment, one of the slits is wider than the other, so that the amplitude of the light from one slit is double that from the other slit. If I_m be the maximum intensity, the resultant intensity when they interfere at phase difference ϕ is given by

(a)
$$\frac{I_m}{3} \left(1 + 2\cos^2\frac{\phi}{2} \right)$$

(b)
$$\frac{I_m}{5} \left(1 + 4\cos^2\frac{\phi}{2} \right)$$

(c)
$$\frac{I_m}{9} \left(1 + 8\cos^2\frac{\phi}{2} \right)$$

(d)
$$\frac{I_m}{9} \left(8 + \cos^2\frac{\phi}{2} \right)$$

6. A point luminous object (O) is at a distance h from front face of a glass slab of width d and of refractive index n. On the back face of slab is a reflecting plane mirror. An observer sees the image of object in mirror (see Fig. (20)).



Figure 20: Problem 6

Distance of image from front face as seen by observer will be

- (a) $h + \frac{2d}{n}$
- (b) 2h + 2d
- (c) h + d

(d)
$$h + \frac{a}{2}$$

- (a) n + n
 7. A uniform wire of diameter 0.04 cm and length 60 cm made of steel (density 8000 kg·m⁻³) is tied at both ends under a tension of 80 N. Transverse vibrations
 - of frequency about 700 Hz will be predominant if the wire is plucked at
 - (a) 15 cm and held at 30 cm.
 - (b) 10 cm and held at 20 cm.
 - (c) 30 cm.
 - (d) 20 cm and held at 40 cm.
- 8. Consider a circle of radius R. A point charge lies at a distance a from its centre and on its axis such that $R = a\sqrt{3}$. If electric flux passing through the circle is ϕ then the magnitude of the point charge is
 - (a) $\sqrt{3\epsilon_0\phi}$
 - (b) $2\epsilon_0\phi$
 - (c) $4\epsilon_0\phi/\sqrt{3}$
 - (d) $4\epsilon_0\phi$
- 9. A uniform tube 60 cm long, stands vertically with lower end dipping into water. When its length above water is 14.8 cm and successively again when it is 48 cm, the tube resonates to a vibrating tuning fork of frequency 512 Hz. The lowest frequency to which this tube can resonate when it is taken out of water is nearly
 - (a) 275 Hz
 - (b) 267 Hz
 - (c) 283 Hz
 - (d) 256 Hz

- 10. A binary star has a period (T) of 2 earth years while distance L between its components having masses M_1 and M_2 is four astronomical units. If $M_1 = M_S$ where M_S is the mass of Sun, the mass of other component M_2 is
 - (a) $3M_S$
 - (b) 7 M_S
 - (c) 15 M_S
 - (d) M_S

Note: The earth - sun distance is one astronomical unit.

11. A uniform rod of mass 2M is bent into four adjacent semicircles each of radius r all lying in the same plane (see Fig. (21)). The moment of inertia of the bent rod about an axis through one end A and perpendicular to plane of rod is



Figure 21: Problem 11

- (a) $22Mr^2$
- (b) $88Mr^2$
- (c) $44Mr^2$
- (d) $66Mr^2$
- 12. Two pulses on the same string are described by the following wave equations:

$$y_1 = \frac{5}{(3x-4t)^2+2}$$
 and $y_2 = \frac{-5}{(3x+4t-6)^2+2}$.

Choose the INCORRECT statement.

- (a) Pulse y_1 and pulse y_2 travel along +ve and -ve x axis respectively.
- (b) At t = 0.75 s, displacement at all points on the string is zero.
- (c) At x = 1 m displacement is zero for all times.
- (d) Energy of string is zero at t = 0.75 s.
- 13. A ray of light enters at grazing angle of incidence into an assembly of five isosceles right-angled prisms having refractive indices μ_1 , μ_2 , μ_3 , μ_4 and μ_5 respectively (see Fig. (22)).



Figure 22: Problem 13

The ray also emerges out at grazing angle. Then

- (a) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 1 + \mu_2^2 + \mu_4^2$
- (b) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 2 + \mu_2^2 + \mu_4^2$
- (c) $\mu_1^2 + \mu_3^2 + \mu_5^2 = \mu_2^2 + \mu_4^2$
- (d) none of the above
- 14. The circuit shown in Fig. (23)) is allowed to reach steady state and then a soft iron core is quickly inserted in the coil such that the coefficient of self inductance changes from L to nL.



The current in the circuit at the time of complete insertion is

- (a) E/R
- (b) nE/R
- (c) E/nR
- (d) zero
- 15. Consider an infinitely extending gas cloud in space with two "rigid" spherical vacuum cavities (see Fig. (24)).



Consider only gravitational forces between gas molecules. Then

- (a) the cavities would come closer to each other.
- (b) the cavities would move away from each other.
- (c) the cavities would be static.
- (d) the motion of cavities would depend on the size of cavities.

Questions (16) and (17) are based on Fig. (25) and following information.



Figure 25: Problems 16 and 17

A rod of mass m and length l is hinged at one end O. A particle of mass m travelling with speed v collides with the rod at a distance x from the centre of mass of the rod such that the reaction force at the hinge is zero.

- 16. Then for the system
 - (a) linear momentum is conserved.
 - (b) angular momentum is not conserved about point O.
 - (c) Linear momentum is not conserved and angular momentum about point *O* is conserved.
 - (d) the mechanical energy is conserved.
- 17. Then
 - (a) x = l/6.
 - (b) x = l/2.
 - (c) x = l/3.
 - (d) x = l/4.
- 18. Consider a huge charge reservoir at potential V. A spherical capacitor C_1 is brought in contact with the charge reservoir and then removed. Next another spherical capacitor C_2 is brought in contact with C_1 and removed. We repeat this process a large number of times. Assume that potential of reservoir does not change during this exercise. Then the charge on C_2 after a very long time is
 - (a) $C_2 V$
 - (b) $C_1 V$
 - (c) $C_2 C_1 V / (C_1 + C_2)$
 - (d) $(C_1 + C_2)V$
- 19. A particle of mass 1 kg is taken along the path *ABCDE* from A to E (see Fig. (26)). The two "hills" are of heights 50 m and 100 m and the horizontal distance AE is 20 m while the path length is 400 m. The coefficient of friction of the surface is 0.1. Take $g = 10 \text{ m} \cdot \text{s}^{-2}$ and $\sqrt{3} = 1.73$. The minimum work on the mass required to accomplish this is



Figure 26: Problem 19

- (a) 20 J
- (b) 173 J
- (c) 400 J
- (d) 0 J
- 20. Two positrons (e^+) and two protons (p) are kept on four corners of a square of side a as shown in Fig. (27). The mass of proton is much larger than the mass of positron. Let q denote the charge on the proton as well as the positron. Then the kinetic energies of one of the positrons and one of the protons respectively after a very long time will be



21. An electrostatic field line leaves at angle α from point charge q_1 , and connects with point charge $-q_2$ at angle β (see Fig. (28)).



Figure 28: Problem 21

Then the relationship between α and β is

- (a) $q_1 \sin^2 \alpha = q_2 \sin^2 \beta$.
- (b) $q_1 \tan \alpha = q_2 \tan \beta$.
- (c) $q_1 \sin^2 \frac{\alpha}{2} = q_2 \sin^2 \frac{\beta}{2}$.
- (d) $q_1 \cos \alpha = q_2 \cos \beta$.
- 22. A square metal frame in the vertical plane is hinged at O at its centre (see Fig. (29)).



A bug moves along the rod PN which is at a distance l from the hinge, such that the whole frame is always stationary, even though the frame is free to rotate in the vertical plane about the hinge. Then the motion of the bug will be simple harmonic, with time period,

- (a) $2\pi\sqrt{l/g}$
- (b) $2\pi\sqrt{2l/g}$
- (c) $2\pi\sqrt{4l/g}$
- (d) $2\pi \sqrt{l/2g}$

[Hint: There is a frictional force between the rod and the bug.]

23. A long flexible inextensible rope of uniform linear mass density λ is being pulled on a rough floor with horizontal force \vec{F} in such a way that its lower part is at rest and upper part moves with constant speed v (see Fig. (30)). The magnitude of \vec{F} will be



Figure 30: Problem 23

- (a) $2\lambda v^2$
- (b) λv^2
- (c) $\lambda v^2/2$
- (d) some function of time and not constant.

- 24. A particle moving with initial velocity $\vec{v}_i = (3\hat{\imath} + 5\hat{\jmath}) \text{ m} \cdot \text{s}^{-1}$ collides with a smooth plane wall placed at some orientation to the particle's trajectory. The resulting velocity of the particle is $\vec{v}_f = (-2\hat{\imath} \hat{\jmath}) \text{ m} \cdot \text{s}^{-1}$. The coefficient of restitution for this collision is
 - (a) 16/33
 - (b) 5/34
 - (c) 16/45
 - (d) 8/45
- 25. A long straight wire is carrying current I_1 in +z direction. The x-y plane contains a closed circular loop carrying current I_2 and not encircling the straight wire. The force on the loop will be
 - (a) $\mu_0 I_1 I_2 / 2\pi$.
 - (b) $\mu_0 I_1 I_2 / 4\pi$.
 - (c) zero.
 - (d) depends on the distance of the centre of the loop from the wire.
- 26. A uniform electric field \vec{E} in the y-direction and uniform magnetic field \vec{B} in the x-direction exists in free space. A particle of mass m and carrying charge q is projected from the origin with speed v_0 along the y-axis. The speed of particle as a function of its y coordinate will be

(a)
$$\sqrt{v_0^2 + \frac{2qEy}{m}}$$

(b) $\sqrt{v_0^2 - \frac{4qEy}{m}}$
(c) $\sqrt{v_0^2 + \frac{qEy}{m}}$
(d) v_0 .

- 27. The atmospheric pressure on the earth's surface is P in MKS units. A table of area 2 m² is tilted at 45⁰ to the horizontal. The force on the table due to the atmosphere is (in newtons)
 - (a) 2P
 - (b) $\sqrt{2}P$
 - (c) $2\sqrt{2}P$
 - (d) $P/\sqrt{2}$
- 28. The shear modulus of lead is 2×10^9 Pa. A cubic lead slab of side 50 cm is subjected to a shearing force of magnitude 9.0×10^4 N on its narrow face (see Fig. (31)).





The displacement of the upper edge is δl , where δl is

- (a) 2×10^{-3} m (b) 5×10^{-4} m (c) 4×10^{-4} m (d) 9×10^{-5} m
- 29. In a moving coil galvanometer the number of turns N = 24, area of the coil $A = 2 \times 10^{-3}$ m², and the magnetic field strength B = 0.2 T. To increase its current sensitivity by 25% we
 - (a) Increase B to 0.30 T
 - (b) Decrease A to $1.5 \times 10^{-3} \text{ m}^2$
 - (c) Increase N to 30
 - (d) None of the above.
- 30. Which of the following statement is TRUE ?
 - (a) Sound waves cannot interfere.
 - (b) Only light waves may interfere.
 - (c) The de Broglie waves associated with moving particles can interfere.
 - (d) The Bragg formula for crystal structure is an example of the corpuscular nature of electromagnetic radiation.
- 31. Two metallic rods AB and BC of different materials are joined together at the junction B (see Fig. (32)). It is observed that if the ends A and C are kept at 100^o C and 0^o C respectively, the temperature of the junction B is 60° C. There is no loss of heat to the surroundings. The rod BC is replaced by another rod BC' of the same material and length (BC = BC'). If the area of cross-section of BC' is twice that of BC and the ends A and C' are maintained at 100^o C and 0^o C respectively, the temperature of the junction Bwill be nearly

(C)HBCSE-TIFR


- 32. Three closed vessels A, B and C are at the same temperature (T) and contain gases which obey the Maxwellian distribution of velocities. The vessel A contains only O_2 , B only N_2 and C a mixture of equal quantities of O_2 and N_2 . If the average speed of the N_2 molecules in vessel B is V_2 and that of oxygen molecules in A is V_1 , the average speed of N_2 molecules in C is
 - (a) $(V_1 + V_2)/2$ (b) $(V_1 - V_2)/2$
 - (c) V_2 (c) V_2
 - (d) $\sqrt{(V_1 V_2)}$
- 33. When a system is taken from state a to state b along the path a c b (see Fig. (33)), 60 J of heat flows into the system and 30 J of work are done by the system. Along the path a d b, if the work done by the system is 10 J, heat flow into the system is



Figure 33: Problem 33

- (a) 100 J
- (b) 20 J
- (c) 80 J
- (d) 40 J

- 34. Two identical piano strings, when stretched with the same tension T_0 , have a fundamental frequency of 300 Hz. The tension in one of the strings is increased to $(T_0 + \Delta T)$ and 3 beats per second occur when both strings vibrate simultaneously. $(\Delta T/T_0) \times 100$ is
 - (a) 2
 - (b) 3
 - (c) 1
 - (d) 4
- 35. The half life of a certain radioactive material $(_zX^{100})$ is 6.93×10^6 s. In order to have an activity of 6.0×10^8 disintegrations per second, the amount of material needed is nearly
 - (a) 10^{-9} kg
 - (b) 10^{-16} kg
 - (c) 10^{-6} kg
 - (d) 10^{-4} kg
- 36. Sound of frequency 1000 Hz from a stationary source is reflected from an object approaching the source at 30 m·s⁻¹, back to a stationary observer located at the source. The speed of sound in air is 330 m·s⁻¹. The frequency of the sound heard by the observer is
 - (a) 1200 Hz
 - (b) 1000 Hz
 - (c) 1090 Hz
 - (d) 1100 Hz
- 37. Current (I) applied voltage (V) characteristics are shown in Fig. (34). Possible observed plot(s) for a photoelectric setup is (are):



Figure 34: Problem 37

- (a) only II.
- (b) I and II.
- (c) II and III.
- (d) II and IV.

- 38. A triply ionized beryllium (Be⁺⁺⁺) has the some orbital radius as the ground state of hydrogen. Then the quantum state n of Be⁺⁺⁺ is
 - (a) n = 1(b) n = 2(c) n = 3(d) n = 4
- 39. One mole of gas undergoes a linear process as shown in Fig. (35).



- (a) Express P in terms of $\{V, V_0, P_0\}$.
- (b) Assuming that the gas is ideal, obtain the expression for T in terms of gas constant R and $\{V, V_0, P_0\}$.
- (c) Obtain the expression for volume change with temperature (dV/dT) in terms of $\{R, V, V_0, P_0\}$.
- (d) Let T_{max} be the maximum temperature in the process. Express T_{max} in terms of $\{V_0, P_0, R\}$.
- (e) Sketch the T V diagram. (T on y-axis and V on x-axis.)
- (f) Let $C_p/C_v = \gamma$, where $C_p(C_v)$ is specific heat at constant pressure (volume). Express heat capacity C_v in terms of R and γ .
- (g) Using the first law of thermodynamics, obtain the expression for specific heat C for the above linear process in terms of $\{R, \gamma, V_0, V\}$.
- (h) Suppose mixture consists of half mole of mono atomic and half mole of diatomic gas. Obtain γ for this mixture.
- (i) For the mixture described in Part (39h), obtain C in terms of $\{R, V, V_0\}$.
- (j) Plot C/R (on y-axis) vs V/V_0 (x-axis).

[13]

400 years of telescope: Dutch eyeglass-maker Hans Lippershey first tried to patent the telescope in October 1608, and his invention was soon a big hit in Europe as a tool for insider trading. Futures contracts were in vogue, and spying a cargo ship first had financial benefits. The telescope also redefined our universe: In 1608, Earth was the centre of God's perfect Creation. By 1610, Galileo showed that Jupiter had moons, Earth's moon had mountains, and the Catholic church was fallible. Four centuries on, we know we are a mere speck in a universe of wonders.



Galileo Galilei (15 Feb.1564-8 Jan.1642) : Italian natural philosopher, astronomer, and mathematician who applied the new techniques of the scientific method to make significant discoveries in physics and astronomy. His great accomplishments include perfecting (though not inventing) the telescope and consequent contributions to astronomy. He studied the science of motion, inertia, the law of falling bodies, and parabolic trajectories. His formulation of the scientific method parallels the writings of Francis Bacon. His progress came at a price, since his ideas were in conflict with religious dogma. He believed the Earth revolved around the Sun. For this, he was interrogated by the Inquisition, was put on trial, found guilty and sentenced to indefinite imprisonment. For renouncing his former beliefs before the Cardinals that judged him, he was allowed to serve this time instead under house-arrest.

"In questions of science the authority of a thousand is not worth the humble reasoning of a single individual."



Chapter II

Brief Solutions

1 INPhO-2006

1. (a) a=0(b) $f_s = 5 \text{ N. and } T \cong 10 \text{ N.}$ 2. (a) $a = \frac{\mu g}{3}$ (b) $t^* = \frac{3}{7} \cdot \frac{R \omega_0}{\mu g}$ (c) $t_{tot} = \frac{R \omega_0}{\mu g}$ 3. (a) $\eta = 1 - \frac{T_c - T_d}{T_b - T_a}$ (b) $\eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ (c)



(d)
$$\eta_c = 1 - \frac{T_d}{T_b}$$
,
Carnot cycle has higher efficiency than the given cycle

4. Amplitude of the image= 2.78 cm The phase difference = 0 Angle made by the virtual image with the principle axis = 30.26°

(b) Maximum Kinetic Energy $K_{max} = 2.75$ eV De Broglie wavelength $\lambda_{dB} = 7.4$ Å

8. (a)
$$y \ge 10^{-18}$$

(b)
$$F = \frac{\eta e^2 y^2 r}{3\epsilon_0 m_n}$$

(c)
$$H = 1.8 \times 10^{-17} \text{ s}^{-1}$$

- (d) $V = V_0 e^{3Ht}$
- (e) The numerical value of H does not agree with the known value of Hubble's constant. This is one of the reasons we may discard the Lyttleton Bondi model.

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2 INPhO-2007

1. (a) $\vec{p} = m(a\hat{r} + r\omega\hat{\theta})$ $\vec{F} = m(-r\omega^2\hat{r} + 2a\omega\hat{\theta})$

(b)
$$\Delta W = \frac{m\omega^2 r}{2}$$

(c) Trajectory will be a spiral.

2. (a)
$$\theta_m = \frac{\cot^{-1}(2\mu)}{2}$$

(b) Possible range of
$$\theta_m$$
: $\theta_m \in]0, \pi/4[$

3. (a)
$$t \approx 222$$
 s

- (b) t = 166 s
- (c) Process (3a) takes more time since as the disk heats up, its specific heat also increases and more heat is required to effect a further rise in temperature.

4. (a)
$$d^2 = R^2 \left(1 - \frac{(n^2 - 2)^2}{4} \right)$$

(b) The allowed range of n is $n\in]\sqrt{2},2[$

5. (a)
$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

(b)
$$V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{a}\right)$$

- (c) $\left(x d\frac{k^2 + 1}{k^2 1}\right)^2 + y^2 = \frac{4d^2k^2}{(k^2 1)^2}$ This is an equation of a circle with centre at $\left(d\frac{k^2 + 1}{k^2 1}, 0\right)$ and radius given by $\frac{2dk}{|k^2 1|}$. The constant k is related to the "equipotential" V_0 .
- (d) See Fig. (2) and part (c)



Figure 2: Problem 5 (d)

(e)
$$v = 1/\sqrt{\epsilon_0 \mu_0} = c$$

6. (a) $F = \frac{\mu_0 I_1 I_2}{\sqrt{3}\pi} \left[\frac{\sqrt{3}S}{2a} - \ln\left(1 + \frac{\sqrt{3}S}{2a}\right) \right]$ and direction downwards.



- ii. Wood : no appreciable change.
- iii. Copper: decrease.

3 INPhO-2008





5. (a) Position of the image with violet centre $f_V = 14.3 \text{ cm}$; Position of the image with red centre $f_R = 16.7 \text{ cm}$; Size of the image with violet centre $I_V \simeq 0.64 \text{ mm}$; Size of the image with red centre $I_R \simeq 0.74 \text{ mm}$

6. (a)
$$\langle F_e \rangle = \frac{C_1^2 V_0^2}{4 \pi \epsilon_0 a^2}$$

(b) $\langle F_m \rangle = \frac{\mu_0 b}{2 h} C_2^2 V_0^2 (2 \pi f)^2$

(c)
$$c = (2\pi)^{3/2} a \left(\frac{b}{h}\right)^{1/2} \frac{C_2}{C_1} f$$

(d) $c = 2.99 \times 10^8 \text{ m} \cdot \text{s}^{-1}$
7. (a) $iR + L \frac{di}{dt} = -N\pi a^2 B_0 \omega \cos \omega t$
(b) $i = \frac{N\pi a^2 B_0 \omega (R \cos \omega t + \omega L \sin \omega t)}{R^2 + \omega^2 L^2}$
(c) $\frac{dF}{dl} = -\frac{NB_0^2 \pi a^2 \omega}{R^2 + \omega^2 L^2} (R \sin \omega t \cos \omega t + \omega L \sin^2 \omega t)$
 $\frac{dF}{dl}\Big|_{av} = -\frac{NB_0^2 \pi a^2 \omega^2 L}{2(R^2 + \omega^2 L^2)}$
 $\frac{dF}{dl}\Big|_{av} = -\frac{NB_0^2 \pi a^2 \omega}{2(R^2 + \omega^2 L^2)} (R \sin 2\omega t - \omega L \cos 2\omega t)$
(d) $\frac{dF}{dl}\Big|_{av} = 1.55 \text{ N} \cdot \text{m}^{-1}$
(e) i. The frequency of the sound is 120 Hz.
ii. The compressional force is lessened and may even become tensile.

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(j)







(a) Let m_1 go down. Then from free body diagram for m_1 ,

$$m_1 g \sin \theta - f - T = m_1 a \tag{1}$$

where the frictional force $f = \mu N$, N being the normal reaction. Hence,

$$m_1 g \sin \theta - \mu N - T = m_1 a$$

 $N = m_1 g \cos \theta$

herefore
$$a = g \sin \theta - \mu g \cos \theta - \frac{T}{m_1}$$
 (2)

From free body diagram for m_2 ,

$$T - m_2 g = m_2 a \tag{3}$$

Solving Eqs. (2) and (3),

Т

$$a = \frac{g \left[m_1(\sin\theta - \mu\cos\theta) - m_2 \right]}{m_2 + m_1}$$

As $m_1 = m_2$,

$$a = \frac{g\left[\sin\theta - 1 - \mu\cos\theta\right]}{2}$$

We can see from the above equation that a can have only negative values. Substituting the values



Figure 2: Free body diagrams assuming m_2 is going down: (a) For m_1 . (b) For m_2 .

Let m_2 go down. Then for m_2

$$m_2g - T = m_2a \tag{4}$$

For m_1

$$T - m_1 g \sin \theta - f = m_1 a \tag{5}$$

Elementary algebraic manipulations yield

$$a = \frac{g \left[1 - \sin \theta - \mu \cos \theta\right]}{2}$$

Substituting the values

 $a = -0.1 \text{ m} \cdot \text{s}^{-2} < 0$

As seen from both the cases, the acceleration is negative, either assuming m_1 going down or m_2 going down. So we can conclude that the acceleration of the system is zero and system would be stationary.

(b) substituting a = 0 in Eq. (4),

$$T \cong 10 \text{ N}.$$

Since system is stationary, so frictional force would be static friction and a may have a positive value only if m_2 is going down and not up. By substituting the value of T and a in Eq. (5), we obtain

$$f_s = 5.2 \,\mathrm{N}.$$

This f_s is down the inclined plane. Why?

2.



Figure 3: Free body diagram of the block disk system

Key points to remember are, when disk touches the block, block-disk system starts to slide. After some time t^* , disk starts to roll without slipping and after this time t^* , disk continues to roll without slipping and comes to a halt in total time t_{tot} .

(a) If a is the acceleration of the block-disk system, then by free body diagram(see Fig. (3)).

$$(2M+M)a = f_1 - f_2$$

$$3Ma = \mu(2M)g - \mu Mg$$

$$a = \frac{\mu g}{3}$$
(6)

(b) As disk rolls, it's angular velocity ω decreases. If α is the angular acceleration of the system, then

$$I \alpha = -f_1 R$$

$$\frac{1}{2} (2M) R^2 \alpha = -\mu (2M) g R$$

$$\alpha = \frac{-2\mu g}{R}$$
(7)

At time t^* , disk starts to roll without slipping. So

$$v(t^*) = R\,\omega(t^*) \tag{8}$$

and equation of motion for rotational motion

$$\omega(t^*) = \omega_0 - \frac{2\,\mu\,g}{R}\,t^* \tag{9}$$

and equation of motion for translational motion

$$v(t^*) = 0 + \frac{\mu g}{3} t^* \tag{10}$$

By Eqs. (9) and (10)

$$t^* = \frac{3}{7} \cdot \frac{R\,\omega_0}{\mu\,g} \tag{11}$$

(c) After time $t > t^*$, velocity of block - disk system starts to decrease. In this condition of pure rolling, frictional force on disk $f'_1 < \mu(2M) g$. Suppose angular acceleration is α' and linear acceleration is a', then

$$\alpha' = \frac{a'}{R}$$

$$I \alpha' = -f_1' R$$

$$\frac{1}{2} (2M) R^2 \alpha' = -f_1' R$$

$$-f_1' = M a'$$
(12)

Hence

Now

$$(2M+M)a' = f'_1 - f'_2$$

$$3Ma' = -Ma' - \mu Mg \Rightarrow a' = \frac{-\mu g}{4}$$
(13)

Suppose in time t from t^* the system comes to rest. Then

$$v(t) = v(t^*) + a't$$

Using Eqs. (10) and (13)

$$0 = \frac{\mu g}{3} \cdot \frac{3R\omega_0}{7\mu g} - \frac{\mu g}{4} t$$
$$= \frac{R\omega_0}{7} - \frac{\mu g t}{4}$$
$$t = \frac{R\omega_0}{\mu g} \frac{4}{7}$$
Thus total time
$$t_{tot} = t + t^*$$
$$t_{tot} = \frac{R\omega_0}{\mu g}$$

This is a surprisingly simple answer.



Figure 4: Plot of force of friction with time

Figure (4) gives the interesting plot of force of friction (f_1) with time. One can make an equivalent plot with angular velocity ω on the x-axis. There is a sudden drop in f, at $t = t * = 3r\omega_0/7\mu g$. This drop occurs when pure rolling sets in. It may remind the reader of a first order phase transition where f_1 plays the role of the internal energy and ω that of temperature.

3. (a) Efficiency of cycle is

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

In the cycle, energy is absorbed in isobaric process from a to b. So $Q_{in} = C_p (T_b - T_a)$. Here C_p is the heat capacity at constant pressure. Energy is released in isobaric process c to d. So $Q_{out} = C_p (T_c - T_d)$. Hence

$$\eta = 1 - \frac{T_c - T_d}{T_b - T_a} \tag{14}$$

Another method exists for this solution. In this method

 $\eta = \frac{W}{Q_{in}}$

and W is evaluated by integrating the area under the P - V curve.



(b) For adiabats b to c and d to a

$$PV^{\gamma} = \text{const.}$$
 (15)

Equation of state gives

$$PV = nRT \Rightarrow \frac{PV}{T} = \text{const.}$$

Inserting this in Eq. (15)

$$T = \text{const.} \times P^{\frac{\gamma - 1}{\gamma}} \tag{16}$$

Equation (16) gives

$$\frac{T_c}{T_d} = \frac{T_b}{T_a} \tag{17}$$

and

$$\frac{T_d}{T_a} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Equation (14) becomes

$$\eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

(c) For isobaric process, V - T curve will be a straight line passing through the origin of the V - T plot. Since $P_1 > P_2$ so for a - b, this line will be less steeper then c - d. By Eq. (15) and equation of state, we can get $T_b > T_c > T_a > T_d$. For convexity of curve from a - d and b - c, we can analyze relation between V and T, that slope is inversely proportional to T. So on this basis we can draw V - T diagram as shown in Fig. (6).



Figure 6: V-T diagram

(d) Carnot efficiency is defined by the highest temperature T_b and lowest temperature T_d :

$$\eta_c = 1 - \frac{T_d}{T_b}$$

Efficiency of concerned cycle (from Eqs. (14) and (17)) is

$$\eta = 1 - \frac{T_d}{T_a}$$

Hence

$$\frac{\eta_c}{\eta} = \frac{1 - \frac{T_d}{T_b}}{1 - \frac{T_d}{T_a}}$$

Since $T_b > T_a$, hence $\eta_c > \eta$ i.e. Carnot engine has higher efficiency than the concerned cycle.

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Figure 7: Problem 4

Given $\mu_1 = 1.0$, $\mu_2 = 1.5$, $\mu_3 = 1.2$, u = -10 cm, R = -10 cm.

General formula for thin lens,

Therefore

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2}$$
R.

Now $R_1 \longrightarrow \infty$, $R_2 = R$.

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_3 - \mu_2}{R} \tag{18}$$

Implies
$$v = \frac{Ru\mu_3}{u(\mu_3 - \mu_2) + \mu_1 R}$$
 (19)
= -17.14 cm

Here the negative sign indicates that image is on the left side. Under small shift along the principal axis of the lens for the object, the image will also shift slightly along the same axis. So, for longitudinal magnification differentiate Eq. (19) with respect to v and get

$$\frac{dv}{du} = \left(\frac{v}{u}\right)^2 \frac{\mu_1}{\mu_3} \\ = 2.4$$

During the first refraction, ray travels from medium $1(\mu_1)$ to medium $2(\mu_2)$. Linear lateral magnification in first refraction

$$\frac{y'_2}{y_1} = \frac{\mu_1}{\mu_2} \frac{v'}{u} \tag{20}$$

For the second refraction

$$\frac{y_2}{y'_2} = \frac{\mu_2}{\mu_3} \frac{v}{v'}$$
(21)

Eq. (20) and Eq. (21) gives

$$\frac{y_2}{y_1} = \frac{\mu_1}{\mu_3} \frac{v}{u}$$
$$= 1.43$$

It is given that object's amplitude is $\sqrt{2}$ cm. Taking the projection of object along the principle axis and perpendicular to the axis,

$$du = 1 \text{ cm (along the axis)}$$

and $y_1 = 1 \text{ cm (normal to the axis)}$
$$\implies dv = 2.4 \text{ cm}$$

and $y_2 = 1.4 \text{ cm.}$
Amplitude of the image $= \sqrt{(dv)^2 + y_2^2}$
 $= 2.78 \text{ cm}$

Since lateral and longitudinal magnifications are positive, the phase difference between oscillating image and object will be zero. Only orientation of the axis along which oscillation takes place will be different.

Angle made by the virtual image with the principle axis



Figure 8: Problem 5

(a) Dipole moment due to a circular ring (r, r + dr) (See Fig. (8))

$$d\mu' = di A$$
And
$$di = \frac{dQ}{T} = \frac{2\pi r \, dr \, \sigma}{2 \, \pi / \omega}$$

$$d\mu' = \frac{2\pi r \, dr \, \omega \, \sigma}{2\pi} \, . \, \pi r^2$$
Then for circular disk
$$\mu' = \pi \omega \sigma \int_0^R r^3 \, dr$$

$$\mu' = \frac{\pi \omega \sigma R^4}{4}$$

Direction is in the positive x-direction.

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5.

(b) Magnetic field due to a circular ring (r, r + dr)

$$dB = \frac{\mu_0}{2} \cdot \frac{di r^2}{(r^2 + a^2)^{3/2}}$$
$$B = \frac{\mu_0}{2} \int_0^R \frac{di r^2}{(r^2 + a^2)^{3/2}}$$

Using Eq. (22)

$$B = \frac{\mu_0 \,\omega \,\sigma}{2} \int_0^R \frac{r^3}{(r^2 + a^2)^{3/2}} \,dr$$

Substitutions

$$r = a \tan \theta$$
$$dr = a \sec^2 \theta \, d\theta$$

and limit 0 to $\tan^{-1}\left(\frac{R}{a}\right)$

$$B = \frac{\mu_0 \,\omega \,\sigma \,a}{2} \int_0^{\tan^{-1}(\frac{R}{a})} \tan^2 \theta \,\sin \theta \,d\theta$$
$$= \frac{\mu_0 \,\omega \,\sigma \,a}{2} \int_0^{\tan^{-1}(\frac{R}{a})} (\sec^2 \theta - 1) \sin \theta \,d\theta$$
$$= \frac{\mu_0 \,\omega \,\sigma \,a}{2} \int_0^{\tan^{-1}(\frac{R}{a})} (\sec^2 \theta \,\sin \theta - \sin \theta) \,d\theta$$

Integrating the first term by parts

$$B = \frac{\mu_0 \,\omega \,\sigma \,a}{2} \,\left[\sin\theta \,\tan\theta + 2\cos\theta\right]_0^{\tan^{-1}\left(\frac{R}{a}\right)}$$

Applying the limits

$$B = \frac{\mu_0 \,\omega \,\sigma}{2} \,\left[\frac{R^2 + 2 \,a^2}{\sqrt{R^2 + a^2}} - 2 \,a \right] \tag{23}$$

Direction is in the positive x-direction.

(c) Method - I

When a >> R then for axial field due to dipole, we can use the formula

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\mu'}{a^3}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2}{a^3} \cdot \frac{\pi \sigma \,\omega R^4}{4}$$
$$= \frac{\mu_0 \,\sigma \,\omega R^4}{8a^3}$$

Direction is in the positive *x*-direction. Method - II

From Eq. (23)

$$B = \frac{\mu_0 \,\omega \,\sigma}{2} \left[\frac{R^2 + 2 \,a^2}{\sqrt{R^2 + a^2}} - 2 \,a \right]$$

= $\frac{\mu_0 \,\omega \,\sigma}{2} \left[\frac{R^2 + 2 \,a^2}{a} \left(1 + \frac{R^2}{a^2} \right)^{-\frac{1}{2}} - 2 a \right]$
= $\frac{\mu_0 \,\omega \,\sigma}{2} \left[\left(\frac{R^2}{a} + 2 a \right) \left(1 - \frac{R^2}{2a^2} + \frac{3 \,R^4}{8 \,a^4} \right) - 2 a \right]$
= $\frac{\mu_0 \,\omega \,\sigma R^4}{8a^3}$ (24)

Direction is in the positive x-direction.

(d) The force on dipole of dipole moment μ placed at a

$$F = \mu \frac{dB}{dx}$$

After differentiating Eq. (24) with respect to a,

$$F = -\frac{3\,\mu\,\mu_0\,\omega\,\sigma\,R^4}{8\,a^4}$$

Direction is dictated by $\vec{\mu}$.

6.



(a) Initial momentum of photon in the beam $\vec{p}_i = p\hat{j}$. Final momentum of photon in the beam $\vec{p}_f = p \sin 2\theta \,\hat{i} - p \cos 2\theta \,\hat{j}$. x component of momentum cancels by symmetry. Net change in momentum $\Delta \vec{p} = -2p \cos^2 \theta$. The number of photons in the annular region r to $r + \Delta r$ per second $= n \ 2\pi r \, dr$. Magnitude of force on the annular region is $\Delta F = n2\pi r \, dr \times 2p \cos^2 \theta$.

Total force on the sphere is (magnitude)

$$F = \int_{0}^{\delta} 4 n\pi p r \cos^{2} \theta dr$$

$$\sin \theta = \frac{r}{R} \qquad \cos^{2} \theta = 1 - \frac{r^{2}}{R^{2}}$$

$$F = 4\pi np \left[\frac{\delta^{2}}{2} - \frac{\delta^{4}}{4R^{2}} \right] \qquad (25)$$

Direction is upward so $\vec{F} = F\hat{j}$.

(b) Method - I Since $\delta \ll R$, drop δ^4/R^2 term in previous part.

$$F \approx 4\pi n p \frac{\delta^2}{2} \tag{26}$$

If E (or $h\nu$) is the energy of one photon and P the power of the beam (1 kW), then

$$\frac{P}{E} = n\pi\delta^2 \tag{27}$$

Also
$$E = pc$$
 (28)

Using Eqs. (27) and (28) in Eq. (26)

$$F = 2\frac{P}{c}$$

For levitation

$$F = mg$$

 $m = \frac{2P}{gc} = 6.80 \times 10^{-7} \,\mathrm{kg}.$

Method - II



Figure 10: Problem 6 (b) method II

Alternatively one can obtain this without part(a). Area = $\pi \delta^2$. Since $\delta \ll R$, we can assume that A-B-C is flat. Change in momentum per photon = 2p.

No. of photons per unit area per sec. = nTherefore

r

$$F = (n \pi \delta^2) 2 p$$
$$= 2 \pi n p \delta^2$$

This is the same as Eq. (26) and we proceed as in the previous part.

Although the answer to the question ends here, we draw your attention to the following. Let us estimate the size of this 'Al' sphere.

$$m = \frac{4\pi}{3}r^{3}\rho$$

$$r^{3} = \frac{3m}{4\pi\rho}$$

$$= \frac{6.8 \times 10^{-7}}{4 \times 2.7 \times 10^{3}} \times \frac{3}{3.14}$$

$$= \frac{6.8 \times 3}{4 \times 2.7 \times 3.14} \times 10^{-10}$$

$$= 10^{-3} \times \left(\frac{6.8 \times 0.3}{4 \times 2.7 \times 3.14}\right)^{1/3}$$

$$= 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm.}$$

Note that focussing a laser beam to a spot size $r/10 = 4 \times 10^{-5}$ m is easy. In this connection, recall that the "minimum" spot size is dictated by the wavelength of the beam. Laser levitations of polystyrene beads have been demonstrated. Optical tweezers is also a possibility. Thus it is possible to realise the experiment.

7. (a) Using formula

$$E_{n_2} - E_{n_1} = 13.6 Z^2 \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \text{ eV}$$

For Li^{++} , Z=3, and transition from $n_2 = 4$ to $n_1 = 3$.

$$E_4 - E_3 = \Delta E = h\nu = h\frac{c}{\lambda} = 13.6 \times 9\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \text{ eV}$$

Putting the values of h and c,

$$\lambda = 2089 \,\text{\AA}$$

This wavelength belongs to the ultraviolet region of the electromagnetic spectrum.

(b)

$$h\nu = \phi + (\text{K.E.})_{max}$$

Here ϕ is the work function of metal. Putting all the values,

$$(K.E.)_{max} = 2.75 \,\mathrm{eV}$$

Now de Broglie wavelength

$$\lambda_{dB} = \frac{h}{\sqrt{2 m_e (\text{K.E.})_{max}}}$$
$$\lambda_{dB} = 7.4 \text{ Å}$$

8. (a) Equating Energies, (or Pressures, or Forces) for a spherical "charged" cloud

$$\frac{3}{5} \frac{Q^2}{4\pi \epsilon_0 R} = \frac{3}{5} \frac{G M^2}{R}$$
$$Q = Nq = N(-ey)$$
$$M = Nm_p$$
$$y^2 = \frac{m_p^2}{e^2} G 4\pi \epsilon_0$$
$$y = 0.9 \times 10^{-18} \simeq 10^{-18}$$
$$y \ge 10^{-18}$$

Hence

(b) The force on the atom at a distance r from the centre is

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q q}{r^2}$$

$$m \frac{dv}{dt} = m \frac{dv}{dr} \cdot \frac{dr}{dt} = m v \frac{dv}{dr} \quad \text{(chain rule)}$$

$$Q = \frac{\eta}{m_p} \frac{4\pi r^3}{3} q$$

$$q = -e y$$

$$v^2 \propto r^2 \text{ or } v \propto r$$

with constant of proportionality

$$H = \left[\frac{\eta}{3\epsilon_0 m_p^2} e^2 y^2\right]^{\frac{1}{2}}$$
$$H = \frac{ey}{m_p} \left[\frac{\eta}{3\epsilon_0}\right]^{\frac{1}{2}}$$

(c)

$$H = \frac{ey}{m_p} \left[\frac{\eta}{3\,\epsilon_0} \right]^{\frac{1}{2}}$$

Putting right values of η , ϵ_0 , m_p , e and $y = 10^{-17}$ (one order of magnitude larger)

$$H = 1.8 \times 10^{-17} \mathrm{s}^{-1}$$

(d) Method - I

$$\frac{dr}{dt} = Hr$$

$$r = r_0 e^{Ht}$$

$$V = V_0 e^{3Ht}$$

Method - II

$$\frac{dV}{dt} = 3 \cdot \frac{4\pi r^2 dr}{3 dt}$$
$$= 3 \cdot \frac{4\pi r^3}{3} H$$
$$\frac{dV}{V} = 3 H dt$$
$$V = V_0 e^{3Ht}$$

(e) Constant H obtained in part (c) is physically similar to Hubble's constant. Observed value of Hubble's constant is $= 2.3 \times 10^{-18} \text{s}^{-1}$, while obtained H is $= 1.8 \times 10^{-17} \text{s}^{-1}$. Further, experiments do not indicate a difference in the magnitudes of the electron and proton charge. Some theories regarding the nature of the fundamental forces and elementary particles also do not point to a difference.



"This is not right. It is so bad, it is not even wrong!"

-Wolfgang Ernst Pauli (25 Apr.1900-15 Dec.1958) on the solution to a problem provided by a colleague.

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1. (a)

Given
$$r = at$$
, $\theta = \omega t$ (29)
Now $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
Hence $\vec{p} = m(a\hat{r} + r\omega\hat{\theta})$
 $\vec{F} = m\vec{a}$
 $= m[(\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}]$
 $= m(-r\omega^2\hat{r} + 2a\omega\hat{\theta})$

(b)

$$\int F_r dr = -m\omega^2 \int_0^r r dr = \frac{-m\omega^2 r^2}{2}$$
(30)

From Eq.
$$(29)$$

$$\int F_{\theta} r \, d\theta = 2m\omega^2 \frac{r^2}{2} = m\omega^2 r^2 \tag{31}$$

Adding Eqs. (30) and (31)

 $\Delta W = \frac{m\omega^2 r^2}{2}$ We can also obtain the above result by employing the work-energy theo-

- (c) Trajectory will be a spiral.
- 2. (a) From Fig. (11)

rem.

$$v_y = v \sin \theta \tag{32}$$

Note that the surface is rough and there is frictional force along the xdirection. Hence elastic collision does not constrain the velocity along the x-direction. It implies that the y-component of the velocity $v \sin \theta$ changes only in sign. From Newton's second law along vertical y-direction, change in momentum is given by the linear impulse, which yields:

$$2mv\sin\theta = \int N\,dt\tag{33}$$

From Newton's second law along x-direction, we have

$$mv\cos\theta - \mu \int N\,dt = mv_x \tag{34}$$

Inserting Eq. (33) in Eq. (34) we obtain

$$v_x = v(\cos(\theta) - 2\mu\sin(\theta))$$



Figure 11: (a) Problem 2. (b) Free body diagram of the ball in contact with the floor.

Range =
$$v_x \times \text{time of flight} = v_x \times \frac{2v_y}{g}$$

 $R(\theta) = \frac{2v^2}{g} f(\theta)$
where
 $f(\theta) = \sin \theta (\cos \theta - 2\mu \sin \theta)$
To maximize R , set
 $f'(\theta_m) = 0$
which yields
 $\theta_m = \frac{1}{2} \cot^{-1}(2\mu)$

(b) Possible range of θ_m :

$$\theta_m \in]0, \pi/4[$$

3. See Fig. (12).

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By Eqs. (35) and (36)

$$\frac{1 - \alpha(\theta_i - \theta_m)}{\theta_m - \theta} \, d\theta - \alpha \, d\theta = \frac{KA}{mC_0 \, L} \, dt$$

Integrating and noting that $mLC_0/KA = 240$ s

$$(1 - \alpha(\theta_i - \theta_m)) \ln\left(\frac{\theta_m - \theta_i}{\theta_m - \theta_f}\right) - \alpha(\theta_f - \theta_i) = \frac{t}{240}$$
(37)

Inserting the values $t \approx 222$ s.

(b) If $\alpha = 0$, then from Eq. (37)

1

$$\frac{t}{240} = \ln\left(\frac{\theta_m - \theta_i}{\theta_m - \theta_f}\right)$$
$$t = 166 \text{ s}$$

(c) Process (3a) takes more time since as the disk heats up, its specific heat also increases and more heat is required to effect a further rise in temperature. Note

$$C(\theta_i) = 600 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$
$$C(\theta_f) = 960 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

4.



$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

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5.

(b) Integrating along a radial line, we get the electric potential

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{a}\right)$$

where a is the distance of reference point from the line charge.

(c) Total potential at point $P \equiv (x, y)$ is (see Fig. (14))

$$V(P) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_+}{r_-}\right)$$

Equipotential lines in z = 0 plane are given by the equation $V(P) = V_0$, some constant. Let $k = \exp(2\pi\epsilon_0 V_0/\lambda)$. Then equation for equipotential lines becomes

$$\frac{r_{-}}{r_{+}} = k$$

$$\frac{(x+d)^{2} + y^{2}}{(x-d)^{2} + y^{2}} = k^{2}$$

$$\left(x - d\frac{k^{2} + 1}{k^{2} - 1}\right)^{2} + y^{2} = \frac{4d^{2}k^{2}}{(k^{2} - 1)^{2}}$$

This is an equation of a circle with centre at $\left(d\frac{k^2+1}{k^2-1},0\right)$ and radius given by $\frac{2dk}{|k^2-1|}$.

(d) See Fig. (14)



Figure 14: Problem 5 (d)

(e) Magnitude of electric force per unit length is given by

$$F_E = \frac{\lambda^2}{4\pi\epsilon_0 d}$$

Magnitude of magnetic force per unit length is given by

$$F_M = \frac{\mu_0 \lambda^2 v^2}{4\pi d}$$

If these are equal then $v = 1/\sqrt{\epsilon_0 \mu_0} = c$. This emphasizes the general observation that magnetic force is very small compared to the electrostatic force.

6.



Figure 15: Problem 6 (a)

(a) See Fig. (15). Force F_1 on side OA is

$$F_1 = \frac{\mu_0 I_2 I_1 S}{2\pi a}$$

downwards, i.e. the negative y - direction.

Consider element dr at a distance r from O on OB. On both of the sides (OB and AB) same magnitude of force acts. The x components cancel and the y components add. Hence total force on both sides

$$F_{2y} = \frac{\mu_0 I_1 I_2}{2\pi} \int_0^S \frac{1}{a + \sqrt{3} r/2} dr$$

upwards, i.e. the positive y - direction.

By adding these forces, total force on the triangle is

$$F = \frac{\mu_0 I_1 I_2}{\sqrt{3}\pi} \left[\frac{\sqrt{3}S}{2a} - \ln\left(1 + \frac{\sqrt{3}S}{2a}\right) \right]$$

This net force will be downwards.

(b) See Fig. (16). The behaviour is quadratic for small S/a and almost linear for large S/a.



Figure 16: Problem 6 (b)

(C)HBCSE-TIFR

7. (a) Let n be the number of photons per second and P, the power of the source.

Change in momentum of one photon

$$= \frac{h}{\lambda}(\hat{\jmath} - \hat{\imath})$$

Rate of change of linear momentum of the beam is

$$\vec{F} = \frac{nn}{\lambda}(\hat{j} - \hat{i})$$
$$= \frac{P}{c}(\hat{j} - \hat{i}) \qquad \left(\because P = \frac{nhc}{\lambda}\right)$$

Inserting values

$$|\vec{F}| = 2.82 \times 10^{-7} \,\mathrm{N}$$

(b) Surface charge density σ of metal surface after time t with photoelectric efficiency η .



Now t = 10 s and $\eta = 0.1$. Hence

$$\sigma = 7.27 \times 10^2 \,\mathrm{C \cdot m^{-2}}$$

(c) Energy density after 10 s

$$= \frac{1}{2} \epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2$$
$$= 2.93 \times 10^{16} \,\mathrm{J} \cdot \mathrm{m}^{-3}$$

(d) Energy of one photon

$$E = \frac{hc}{\lambda} = 2.06 \,\mathrm{eV}$$

Given that work function

$$\phi = 1.9\,\mathrm{eV}$$

Then

 $\mathbf{K}_{max} = \mathbf{E} - \phi = 0.16 \text{ eV}$

Hence the range of kinetic energy is from 0 to 0.16 eV.

8.



Figure 17: Problem 8

(a) From Fig. (17)

$$\Delta p = -\rho g \Delta z$$

Let m_a be the mass of one mole of air. Then by ideal gas law

$$p = \frac{\rho RT}{m_a}$$

Also, the equation for adiabatic process is $p^{\gamma-1} \propto T^{\gamma}$ From these equations,

$$\Delta T = -\frac{(\gamma - 1)}{\gamma} \frac{m_a g}{R} \Delta z$$

The lapse rate equation is

$$\Gamma = -\frac{(\gamma - 1)}{\gamma} \frac{m_a g}{R}$$

- (b) For diatomic gas $\gamma = 7/5$ and $m_a = 2.9 \times 10^{-2} \text{ kg} \cdot \text{mol}^{-1}$. Hence $\Gamma = -0.01 \text{ K} \cdot \text{m}^{-1}$ (i.e. 1⁰ C decrease for every 100 m).
- (c) Using equations of part (8a)

$$\frac{dp}{p} = -\frac{m_a g}{R} \frac{dz}{(T_0 - \Gamma z)}$$

On integrating

$$p = p_0 \left(\frac{T_0 - \Gamma z}{T_0}\right)^{m_a g/RI}$$

There are two interesting aspects about the exponent of the above equation. The first is a rare occurance of "Force" related dimension in gases, namely $R\Gamma$. The second is its numerical value $m_a g/R\Gamma \simeq 3$. For reasonable heights (e.g. z = 1 km)

$$p(z) \simeq p_0 \left[1 - \frac{m_a g z}{RT_0} \right]$$

- (d) Inserting given values in the above equation, the height of the atmosphere is approximately 30 km.
- 9. (a) The magnetic field at the centre of the field coil is

$$B = \frac{\mu_0}{2} \frac{N_f}{R_f} I(t)$$

(b) Flux on the pick-up coil

$$\phi = \pi R_p^{2} B$$

Induced emf in N_p turns

$$\varepsilon = -N_p \frac{d\phi}{dt}$$

$$\varepsilon| = \frac{\mu_0}{2} \frac{N_p N_f}{R_f} \pi R_p^2 \frac{dI(t)}{dt}$$

(C)HBCSE-TIFR
(c) For maximum ε

$$\varepsilon_0 = \frac{\mu_0}{2} \frac{N_p N_f}{R_f} \pi R_p^2 I_0 \omega$$

$$\therefore \quad N_p N_f = 645 \tag{38}$$

(d) Mutual inductance on field coil due to pick up coil is equal to mutual inductance on pick up coil due to field coil. Hence mutual inductance

$$M = \frac{\mu_0}{2} \frac{N_p N_f}{R_f} \pi R_p^2 = 1.59 \times 10^{-5} \,\mathrm{H}$$

(e) The length of wire used is

$$L = 2\pi R_f N_f + 2\pi R_p N_p$$

To optimize it

$$\frac{dL}{dN_f} = 0$$

1 1

Eq. (38) yields

$$N_f = 18 \,\mathrm{turns}, \ N_p = 36 \,\mathrm{turns}$$

- (f) Induced emf in case of
 - i. Iron : will increase.
 - ii. Wood : no appreciable change.
 - iii. Copper: decrease.



Max Planck (23 Apr.1858-4 Oct.1947) : Max (Karl Ernst Ludwig) Planck was a German theoretical physicist. He studied at Munich and Berlin, where he studied under Helmholtz, Clausius and Kirchoff and subsequently joined the faculty. He became professor of theoretical physics (1889-1926). His work on the law of thermodynamics and the distribution of radiation from a black body led him to abandon classical Newtonian principles and introduce the quantum theory (1900). For this he was awarded the Nobel Prize for Physics in 1918.

This assumes that energy is not infinitely divisible, but ultimately exists as discrete amounts he called quanta (Latin, "how much"). Further, the energy carried by a quantum depends in direct proportion to the frequency of its source radiation. The work leading to the "lucky" blackbody radiation formula was described by Planck in his Nobel Prize acceptance speech (1920): "But even if the radiation formula proved to be perfectly correct, it would after all have been only an interpolation formula found by lucky guess-work and thus, would have left us rather unsatisfied. I therefore strived from the day of its discovery, to give it a real physical interpretation and this led me to consider the relations between entropy and probability according to Boltzmann's ideas. After some weeks of the most intense work of my life, light began to appear to me and unexpected views revealed themselves in the distance." [See also the Foreword]

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1. Note that

 $A{:}$ has dimensions of energy.

 $B\colon$ has dimensions of length.

C: is dimensionless.

Now the Bohr radius will have a combination of B and C.

$$r_n = \frac{n^2}{2\pi} \frac{B}{C}$$

The energy will have a combination of A and C.

$$E_n = -\frac{1}{2n^2}AC^2$$

The Rydberg constant (of dimensions length inverse) will have a combination of B^{-1} and C.

$$R = \frac{C^2}{2B}$$

Note: This problem can be done in number of ways.

2.

At
$$t = 0, x = 0, y = H$$

 $v_x = u, v_y = 0$
H
H
y
x
Figure 18: Problem 2 (a)
(a) See Fig.(18)

$$\frac{dv_x}{dt} = -cv_x$$

$$x = \frac{u}{c}(1 - e^{-ct})$$

$$\frac{dv_y}{dt} = -g - cv_y$$

$$\frac{dv_y}{dt} = -g - cv_y$$

$$\frac{dy}{dt} = -\frac{g}{c}(e^{-ct} - 1)$$
and $y = H + \frac{g}{c^2} - \frac{g}{c} \left[\frac{1}{c}e^{-ct} + t\right]$
(40)

(C)HBCSE-TIFR

(b) From Eq. (39)

$$t = -\frac{1}{c}\ln\left(1 - \frac{xc}{u}\right)$$

Substituting in Eq. (40),

$$y = H + \frac{g}{c^2} - \frac{g}{c} \left[\frac{1}{c} e^{\ln\left(1 - \frac{xc}{u}\right)} - \frac{1}{c} \ln\left(1 - \frac{xc}{u}\right) \right]$$
$$y = H + \frac{g}{c^2} - \frac{g}{c^2} \left(1 - \frac{xc}{u}\right) + \frac{g}{c^2} \ln\left(1 - \frac{xc}{u}\right)$$
$$y = H + \frac{gx}{cu} + \frac{g}{c^2} \ln\left(1 - \frac{xc}{u}\right)$$

If c is small and the range is limited, e.g. $\frac{xc}{u} \ll 1$ then

$$y = H + \frac{gx}{cu} + \frac{g}{c^2} \left[-\frac{xc}{u} - \frac{x^2c^2}{2u^2} - \frac{x^3c^3}{3u^3} + \cdots \right]$$
(41)
$$y = H - \frac{gx^2}{2u^2} - \frac{gx^3c}{3u^3}$$

(c) The trajectory is foreshortened (see Fig. (19)). Note c = 0 is without air resistance and $c \neq 0$ is with air resistance.



Figure 19: Problem 2 (c)

(d) When y = 0, from Eq. (40)

$$H + \frac{g}{c^2} \left[1 - e^{-ct} - ct \right] = 0$$

For $H = 500$ m, $c = 0.05 \text{ s}^{-1}$, $g = 10 \text{ m} \cdot \text{s}^{-2}$
 $t = 10 + 1.1 = 11.1 \text{ s}$

The above answer is obtained by iterative analysis. You may verify that the approximation in Eq. (41) is valid.

- 3. (a) Free Body Diagram, see Fig. (20)
- ©HBCSE-TIFR



Equating Eqs. (44) and (45) we obtain

$$H = \frac{R}{2} \left[\sqrt{1 + \frac{8 R_g^3}{R^3}} - 1 \right]$$

= 1.51 × 10⁵ km

(d) See Fig. (21)



Hence,

$$q_w = \frac{k_w}{w} \ \frac{(\Delta - \delta)}{2}$$

(c)

$$E_1 = \epsilon \sigma T'^4 + (1 - \epsilon) E_2$$

and

$$E_2 = \epsilon \sigma T''^4 + (1 - \epsilon)E_1$$
$$q_r = E_2 - E_1$$
$$= \frac{\epsilon \sigma}{(2 - \epsilon)} \left(T''^4 - T'^4\right)$$

(d) Using Eq. (46) and given set of equations,

$$T'' = T_0 + \frac{\delta}{2}, \quad T' = T_0 - \frac{\delta}{2}$$
 (47)

also,

$$T''^4 - T'^4 = (T''^2 - T'^2)(T''^2 + T'^2)$$
$$= 2\delta T_0 [(T'' + T')^2 - 2T'' T']$$

putting values from Eq. (47)

$$= 4\delta T_0^3 \left[1 + \left(\frac{\delta}{2T_0}\right)^2 \right]$$

again, $q_r = q_w$

$$\frac{\epsilon \sigma}{(2-\epsilon)} \left(T^{\prime\prime 4} - T^{\prime 4} \right) = \frac{k_w}{w} \frac{(\Delta - \delta)}{2}$$

since, $(\delta^2 << T_0^2)$

$$\frac{\epsilon \sigma}{(2-\epsilon)} 4 \,\delta T_0^3 = \frac{k_w}{w} \,\frac{(\Delta-\delta)}{2}$$
$$1 - \frac{\delta}{\Delta} = 8 \,c \,T_0^3 \,\frac{\delta}{\Delta}$$

where,

$$c = \frac{\sigma \epsilon w}{k_w (2 - \epsilon)}$$
$$\delta = \frac{\Delta}{1 + 8cT_0^3}$$

Now,

$$q_r = q_w = \frac{k_w}{w} \frac{(\Delta - \delta)}{2} = \frac{k_w}{w} \left(\frac{\Delta 4c T_0^3}{1 + 8c T_0^3}\right)$$

(e)

$$c=6.44\times 10^{-10},~w=0.01\,$$
 m, $~k_w=0.72\;{\rm W}{\cdot}{\rm m}^{-1}\cdot{\rm K}^{-1}$
$$T_0=284\;{\rm K},~\Delta=28\,{\rm K}$$

$$q_r=107.22\;{\rm W}{\cdot}{\rm m}^{-2}$$

(f)

$$q_{cv} = \frac{N_u K_a}{d} \left(T'' - T' \right) = \frac{N_u K_a}{d} \delta$$
$$q_{cv} = q_w$$

gives,

$$\delta = \frac{\Delta}{1 + \frac{2 w k_a N_u}{k_w d}}$$
$$q_{cv} = \frac{N_u k_a \Delta k_w}{k_w d + 2 w k_a N_u}$$

(g) Ignoring $2wk_a N_u$ $q_{cv} \simeq 46.5 \text{ W} \cdot \text{m}^{-2}$

(h)

$$q_{cd} = \frac{k_s}{d}(T'' - T') = \frac{k_s \delta}{d}$$
$$q_{cd} = q_w$$

From part (b),

$$\frac{k_w}{2w}(\Delta - \delta) = \frac{k_s \delta}{d}$$

which gives,

$$\delta = \frac{k_w \,\Delta \,d}{2 \,w k_s + k_w \,d}$$

Hence,

$$q_{cd} = \frac{k_s \, k_w \, \Delta}{2 \, w k_s + k_w \, d}$$

- (i) $k_s = 0.05 \text{ W} \cdot \text{m}^{-1} \text{K}^{-1}, k_w = 0.72 \text{ W} \cdot \text{m}^{-2}$ w = 0.01 m, d = 0.1 m $q_{cd} = 13.8 \text{ W} \cdot \text{m}^{-2}$
- (j) Since,

 $q_{cd} < q_{cv} < q_r$

Hence sheathing material is best for insulation.

- 5. See the brief solution.
- 6. (a) The force on one capacitor plate due to the other is

$$F_e = \frac{Q_1^2}{2A\epsilon_0}$$
$$F_e = \frac{C_1^2 V_0^2 \cos^2(2\pi ft)}{2\pi a^2 \epsilon_0}$$

Time - averaged force is

$$\langle F_e \rangle = \frac{C_1^2 V_0^2}{4 \pi \epsilon_0 a^2} \tag{48}$$

$$i_2 = -C_2 V_0 2 \pi f \sin(2\pi f t)$$

 $F_m = i_2 l B$

Hence,

$$F_m = \frac{\mu_0 b}{h} (-C_2 V_0 2\pi f \sin(2\pi f t))^2$$

Time- averaged force is

$$\langle F_m \rangle = \frac{\mu_0 b}{2 h} C_2^2 V_0^2 (2\pi f)^2$$
(49)

(c) Equating Eqs. (48) and (49)

$$\langle F_e \rangle = \langle F_m \rangle$$

and noting $c^2 = \frac{1}{\mu_0 \epsilon_0}$

2 00 × 108

We obtain

$$c = (2\pi)^{3/2} a \left(\frac{b}{h}\right)^{1/2} \frac{C_2}{C_1} f$$
(50)

-1

(d) From Eq. (50) and given constants.

$$\overline{B}$$

$$|\overline{B}| = B_0 \sin \omega t$$

$$|\overline{A}|$$

Figure 23: Problem 7

7. (a) Flux : $\phi = B_0 \sin(\omega t) \pi a^2 N$ $\varepsilon = -\frac{d\phi}{dt} = -N\pi a^2 B_0 \omega \cos \omega t$

From Kirchhoff's Law,

$$iR + L\frac{di}{dt} = -N\pi a^2 B_0 \,\omega \cos \omega t \tag{51}$$

(b) Take $i = Rl(i_0 e^{j\omega t})$ $(j^2 = -1)$ Substituting the complex form in Eq. (51), we obtain,

$$i_0 = \frac{N \pi a^2 B_0 \omega (R - j\omega L)}{R^2 + \omega^2 L^2}$$

This implies,

$$i = \frac{N \pi a^2 B_0 \omega (R \cos \omega t + \omega L \sin \omega t)}{R^2 + \omega^2 L^2}$$

(c) The elemental force

$$d\overline{F}=i\,d\overline{l}\times\overline{B}$$

is directed radially in. Substituting,

$$\frac{dF}{dl} = -\frac{NB_0^2 \pi a^2 \omega}{R^2 + \omega^2 L^2} (R \sin \omega t \, \cos \omega t + \omega \, L \sin^2 \omega t)$$

Time - averaged compressional force

$$\frac{dF}{dl}\Big|_{av} = -\frac{NB_0^2 \pi a^2 \omega^2 L}{2(R^2 + \omega^2 L^2)}$$
(52)

$$\left. \frac{dF}{dl} \right|_{osc} = \left. -\frac{NB_0^2 \pi a^2 \omega}{2(R^2 + \omega^2 L^2)} (R\sin 2\omega t - \omega L\cos 2\omega t) \right.$$
(53)

Net force on ring is zero by symmetry.

(d) From Eq. (52)

$$\frac{dF}{dl}\Big|_{av} = -\frac{\pi (10^{-2}) \, 10^6 \, 10^{-1}}{2(10^2 + 10^4)}$$
$$\simeq -\frac{\pi}{2} 10^{-1} N$$
$$= 1.55 \, \text{N} \cdot \text{m}^{-1}$$

- (e) i. From Eq. (53) the oscillating force has a frequency of 2ω and hence the frequency of the sound is 120 Hz.
 - ii. The inclusion of the capacitor will result in

$$\omega L \rightarrow \omega L - \frac{1}{\omega C}$$

 ωC \therefore the compressional force is lessened and may even become negative, i.e. tensile and outward.



(8 June 1936 -)

"In the fall of 1970 Ben Widom asked me to address his statistical mechanics seminar on the renormalization group. He was particularly interested because Di Castro and Jona-Lasinio had proposed applying the field theoretic renormalization group formalism to critical phenomena, but no one in Widom's group could understand Di Castro and Jona-Lasinio's paper. In the course of lecturing on the general ideas of fixed points and the like I realized I would have to provide a computable example, even if it was not accurate or reliable. I applied the phase space cell analysis to the Landau-Ginzburg model of the critical

point and tried to simplify it to the point of a calculable equation, making no demands for accuracy but simply trying to preserve the essence of the phase space cell picture. The result was a recursion formula in the form of a nonlinear integral transformation on a function of one variable, which I was able to solve by iterating the transformation on a computer. I was able to compute numbers for exponents from the recursion formula at the same time that I could show (at least in part) that it had a fixed point and that the scaling theory of critical phenomena of Widom et *al.* followed from the fixed point formalism. Two papers of 1971 on the renormalization group presented this work."

Kenneth Wilson, Nobel Laureate, 1982 on how a Nobel prize winning work was born from an effort to cook up a simple example. [See also the Foreword]

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(39).

$$P_{0}$$

$$P = mV + C$$
Note that at $P = 0$, $V = V_{0}$. Hence
$$C = -mV_{0}$$

$$m = \frac{-P_{0}}{V_{0}}$$

$$P = \frac{-P_{0}}{V_{0}}(V - V_{0})$$
(54)
Rewriting we obtain
$$\frac{P}{P_{0}} + \frac{V}{V_{0}} = 1 \qquad (P < P_{0}, V < V_{0})$$
(55)
(b) The ideal gas law $(PV = RT)$ for one mole of the gas implies
$$P = \frac{RT}{V}$$
Using Eq.(55),
$$\frac{RT}{V} = -\frac{P_{0}}{V_{0}}(V - V_{0})$$
Rewriting,
$$T = \frac{P_{0}V}{V_{0}}\left(1 - \frac{V}{V_{0}}\right)$$
(56)
(c) From part (b)
$$\frac{RT}{P_{0}} = V\left(1 - \frac{V}{V_{0}}\right)$$
Differentiating with respect to T,

 $\implies \frac{R}{P_0} = \frac{dV}{dT} \left(1 - \frac{V}{V_0} \right) + V \left(-\frac{1}{V_0} \frac{dV}{dT} \right)$

$$= \frac{dV}{dT} \left(1 - \frac{2V}{V_0}\right)$$
Rewriting,

$$\frac{dV}{dT} = \frac{RV_0}{P_0(V_0 - 2V)}$$
(57)
(d) From part (c)

$$\frac{dT}{dV} = \frac{P_0}{R} \left(1 - \frac{2V}{V_0}\right)$$
For maximum temperature,

$$\frac{dT}{dV} = 0$$
This occurs at

$$V = V_0/2$$
Hence

$$T_{max} = \frac{P_0V_0}{4R}$$
(58)
(e)

$$T_{max} = \frac{P_0V_0}{4R}$$
(58)
(f) Note that

$$C_p - C_v = R$$
and

$$C_v = C_p/\gamma$$
Hence

$$C_v = \frac{R}{\gamma - 1}$$
(g) From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

where

$$\triangle U = C_v dT$$

Rewriting the first law,

$$CdT = C_v dT + P dV$$

$$\therefore C = C_v + P \frac{dV}{dT}$$

$$= \frac{R}{\gamma - 1} + P_0 \left(1 - \frac{V}{V_0}\right) \frac{R}{P_0 \left(1 - \frac{2V}{V_0}\right)} \quad \text{(From Eq. (54) and (57))}$$

Rewriting,

R

$$C = \frac{R}{\gamma - 1} + \frac{(V_0 - V)R}{(V_0 - 2V)}$$
(59)

(h) For the mixture of gases the "adiabatic" constant is

$$\gamma = \frac{\sum n_i C_{pi}}{\sum n_i C_{vi}}$$

$$= \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R} \quad (\text{Since } n = 1/2)$$

$$= \frac{3}{2}$$
(i) Using $\gamma = \frac{3}{2}$ in Eq. (59) we obtain
$$C = R \frac{\left(3 - \frac{5V}{V_0}\right)}{\left(1 - \frac{2V}{V_0}\right)}$$

(j) Note that negative specific heat implies that temperature goes down although heat is being pumped into the system. This is on account of the peculiar linear nature of the process. Also note that temperature too decreases for $V > V_0/2$, the students are advised to calculate.





(29 July 1898-11 Jan.1988)

sacred and profound."

"Some of the young people I see, who are very good, take physics... as a system you can do things with, can calculate something with, and they miss... the mystery of it : how very different it is from what you can see, and how profound nature is...There is no good translation of the Yiddish word '*Witz*'. It's a joke or a trick or a sleight of hand. You can always bulldoze your way to an answer, but it's the use of this kind of witty trick or subtle approach that I have always liked about physics.... I have always taken physics personally....It's been me and nature and nature is

-Isidor Isaac Rabi, Nobel Laureate, 1944 on how not to approach a physics problem and how to view nature with respect and awe.

Appendix A

List of Acronyms

HBCSE	Homi Bhabha Centre for Science Education
TIFR	Tata Institute of Fundamental Research
IAPT	Indian Association of Physics Teachers
IACT	Indian Association of Chemistry Teachers
IATBS	Indian Association of Teachers in Biological Sciences
NSE	National Standard Examination
NSEP	National Standard Examination in Physics
NSEC	National Standard Examination in Chemistry
NSEB	National Standard Examination in Biology
INO	Indian National Olympiads
INPhO	Indian National Physics Olympiads
INChO	Indian National Chemistry Olympiads
INBO	Indian National Biology Olympiads
OCSC	Orientation cum Selection Camp
PDT	Pre-departure Training Camp
IPhO	International Physics Olympiad
IChO	International Chemistry Olympiad
IBO	International Biology Olympiad
NCERT	National Council of Education Research and Training
CBSE	Central Board of Secondary Education
IIT-JEE	Indian Institute of Technology - Joint Entrance Examination
AIIMS	All India Institute of Medical Sciences
DAE	Department of Atomic Energy
DST	Department of Science and Technology
MHRD	Ministry of Human Resource Development
BRNS	Board of Research in Nuclear Sciences



Appendix B IPhO and INPhO Syllabi

International Physics Olympiad Syllabus

General

- a. The extensive use of the calculus (differentiation and integration) and the use of complex numbers or solving differential equations should not be required to solve the theoretical and practical problems.
- b. Questions may contain concepts and phenomena not contained in the Syllabus but sufficient information must be given in the questions so that candidates without previous knowledge of these topics would not be at a disadvantage.
- c. Sophisticated practical equipment likely to be unfamiliar to the candidates should not dominate a problem. If such devices are used then careful instructions must be given to the candidates.
- d. The original texts of the problems have to be set in the SI units.

A. Theoretical Part

The first column contains the main entries while the second column contains comments and remarks if necessary.

1. Mechanics

a) Foundation of kinematics of a	Vector description of the position of the point
point mass	mass, velocity and acceleration as vectors
b) Newton's laws, inertial systems	Problems may be set on changing mass
c) Closed and open systems, momen-	
tum and energy, work, power	
d) Conservation of energy, conserva-	
tion of linear momentum, impulse	
e) Elastic forces, frictional forces, the	Hooke's law, coefficient of friction (F/R)
law of gravitation, potential energy	const), frictional forces, static and kinetic,
and work in a gravitational field	choice of zero of potential energy
f) Centripetal acceleration, Kepler's	
laws	

2. Mechanics of Rigid Bodies

a) Statics, center of mass, torque	Couples, conditions of equilibrium of bodies
b) Motion of rigid bodies, translation, ro- tation, angular velocity, angular accelera- tion, conservation of angular momentum	Conservation of angular momentum about fixed axis only
c) External and internal forces, equation of motion of a rigid body around the fixed axis, moment of inertia, kinetic energy of a rotating body	Parallel axes theorem (Steiner's the- orem), additivity of the moment of inertia
d) Accelerated reference systems, inertial forces	Knowledge of the Coriolis force for- mula is not required

3. Hydromechanics No specific questions will be set on this but students would be expected to know the elementary concepts of pressure, buoyancy and the continuity law.

4. Thermodynamics and Molecular Physics

a) Internal energy, work and heat, first and second laws of thermodynamics	Thermal equilibrium, quantities de- pending on state and quantities de- pending on process
b) Model of a perfect gas, pressure and molecular kinetic energy, Avogadro's num- ber, equation of state of a perfect gas, ab- solute temperature	Also molecular approach to such sim- ple phenomena in liquids and solids as boiling, melting etc.
c) Work done by an expanding gas limited	Proof of the equation of the adiabatic
to isothermal and adiabatic processes	process is not required
d) The Carnot cycle, thermodynamic ef- ficiency, reversible and irreversible pro- cesses, entropy (statistical approach), Boltzmann factor	Entropy as a path independent func- tion, entropy changes and reversibil- ity, quasistatic processes

5. Oscillations and waves

a) Harmonic oscillations, equation of har	Solution of the equation for harmonic
a) marinome oscillation, equation of mar-	motion, attenuation and resonance -
mome oscillation	qualitatively

b) Harmonic waves, propa- gation of waves, transverse and longitudinal waves, lin- ear polarization, the clas- sical Doppler effect, sound waves	Displacement in a progressive wave and understanding of graphical representation of the wave, measurements of ve- locity of sound and light, Doppler effect in one dimension only, propagation of waves in homogeneous and isotropic media, reflection and refraction, Fermat's principle
c) Superposition of har- monic waves, coherent waves, interference, beats, standing waves	Realization that intensity of wave is proportional to the square of its amplitude. Fourier analysis is not required but candidates should have some understanding that complex waves can be made from addition of simple si- nusoidal waves of different frequencies. Interference due to thin films and other simple systems (final formulae are not required), superposition of waves from secondary sources (diffraction)

6. Electric Charge and Electric Field

a) Conservation of charge, Coulomb's law	
b) Electric field, potential, Gauss' law	Gauss' law confined to simple symmetric systems like sphere, cylinder, plate etc., elec- tric dipole moment
c) Capacitors, capacitance, dielectric con-	
stant, energy density of electric field	

7. Current and Magnetic Field

a) Current, resistance, internal resis-			
tance of source, Ohm's law, Kirchhoff's	Simple cases of circuits containing non-ohmic		
laws, work and power of direct and al-	devices with known V-I characteristics		
ternating currents, Joule's law			
b) Magnetic field (B) of a current, cur-	Particles in a magnetic field, simple applica-		
rent in a magnetic field, Lorentz force	tions like cyclotron, magnetic dipole moment		
a) Amporala law	Magnetic field of simple symmetric systems like		
c) Ampere's law	straight wire, circular loop and long solenoid		
d) Law of electromagnetic induc-			
tion, magnetic flux, Lenz's law, self-			
induction, inductance, permeability, en-			
ergy density of magnetic field			
e) Alternating current, resistors, induc-	Simple AC circuits, time constants, final for		
tors and capacitors in AC-circuits, volt-	mulae for parameters of concrete recompany of		
age and current (parallel and series) res-	cuits are not required		
onances			

8. Electromagnetic waves

a) Oscillatory circuit, frequency of oscilla-	
tions, generation by feedback and resonance	
b) Wave optics, diffraction from one and	
two slits, diffraction grating, resolving	
power of a grating, Bragg reflection,	
c) Dispersion and diffraction spectra, line	
spectra of gases	
d) Electromagnetic waves as transverse	Superposition of polarized waves
waves, polarization by reflection, polarizers	Superposition of polarized waves
e) Resolving power of imaging systems	
f) Black body, Stefan-Boltzmann law	Planck's formula is not required

9. Quantum Physics

a) Photoelectric effect, energy and impulse of the photon	Einstein's formula is required
b) De Broglie wavelength, Heisenberg's un-	
certainty principle	

10. Relativity

a) Principle of relativity, addition of velocities, relativistic Doppler effect

b) Relativistic equation of motion, momentum, energy, relation between energy and mass, conservation of energy and momentum

11. Matter

a) Simple applications of the Bragg equation

b) Energy levels of atoms and molecules (qualitatively), emission, absorption, spectrum of hydrogen like atoms

c) Energy levels of nuclei (qualitatively), alpha-, beta- and gamma-decays, absorption of radiation, halflife and exponential decay, components of nuclei, mass defect, nuclear reactions

Indian National Physics Olympiad Syllabus

This is broadly equivalent to senior secondary level (Class XI and Class XII) of the Central Board of Secondary Education (CBSE). For example it does not include Fermat's principle and special relativity. Some of the problems are unconventional, of high difficulty level, and comparable to the International Olympiads.

Appendix C

The Stages of the Indian Physics Olympiad Program

The Olympiad programme is a 5 STAGE process for each subject separately. Stage I for each subject is organized by the Indian Association of Physics Teachers (IAPT) with the assistance of Indian Association Chemistry Teachers (IACT) and Indian Association of Teachers in Biological Sciences (IATBS). All the subsequent stages are organized by the Homi Bhabha Centre for Science Education (HBCSE).

- Stage I : National Standard Examinations (NSEs) NSEs are usually conducted in the last week of November at about 1000 centres all over India. Over 35,000 students enroll in the National Standard Examination in Physics (NSEP).
- Stage II : Indian National Olympiad Examinations (INOs) Around 300 meritorious students from NSEs are selected for Indian National Olympiad (INO) examination in each subject. These examinations are usually conducted either in the last week of January or in the first week of February at about 15 centres in the country.
- Stage III : Orientation Cum Salection Camp (OCSC) About 35 students in each subject are chosen on the basis of their performance in INO exams. The selected group of students in each subject are invited to the OCSC for two to three weeks which are usually held in April-June. Five best students in Physics (four in Chemistry and Biology each) are selected to represent India at respective International Olympiads.
- Stage IV : Pre-Departure Training Camp (PDT) The selected Indian teams undergo rigorous training before departing for International Olympiads.
- **Stage V : Participation in International Olympiads** Selected students and 2 to 3 teacher leaders and scientific observers constitute the delegation to represent India at the International Olympiads which are normally held in July.

Information regarding Stage I is available on

IAPT website - http://www.iapt.org.in

and information regarding stages II to V and details of eligibility for various stages are available on

HBCSE website - http://olympiads.hbcse.tifr.res.in.



Appendix D

Performance of The Indian Team

1998

Dates	Place	Student's Name	Place	Medals
		Abhishek Kumar	Bokaro	Silver
July 2, 1998		Vijay Bhat	Kolkata	Bronze
to	Reykjavik,	Shivi Kumar Bansal	Obra (U.P.)	Honourable Mention
July 10, 1998	Iceland	Dilys Thomas	Pune	Honourable Mention
		Saikat Guha	Patna	Honourable Mention

Delegation Leader : Prof. H.C.Pradhan (HBCSE, Mumbai) **Pedagogical Leader** : Prof. T. S. Natrajan (IIT, Chennai)

1999				
Dates	Place	Student's Name	Place	Medals
		Sandeep Bala	Mumbai	Silver
July 18, 1999		Harsh Madhyastha	Bangalore	Silver
to	Padua, Italy	Mayank Rawat	Panchkula	Silver
July 27, 1999		Amit Agarwal	Chandigarh	Silver
		Suvrat Raju	Delhi	Bronze

Delegation Leader : Prof. R. M. Dharkar (IAPT) **Pedagogical Leader** : Prof. Vijay A. Singh (IIT, Kanpur)

2000

Dates	Place	Student's Name	Place	Medals
		Navneet Loiwal	Jaipur	Gold
July 8, 2000		M. Arvind	Chennai	Gold
to	Leicester,	Abhineet Sawa	Rourkela	Silver
July 16, 2000	U.K.	Nipun Kwatra	Chandigarh	Silver
		V. Srikant	Hyderabad	Honourable Mention

Delegation Leader : Prof. Vijay A. Singh (IIT, Kanpur)Pedagogical Leader : Prof. D. A. Desai (D. G. Ruparel College, Mumbai)Scientific Observer : Prof. Arvind Kumar (HBCSE, Mumbai)

Dates	Place	Student's Name	Place	Medals
		Nandan Dixit	Mumbai	Gold
June 28, 2001		Arvind Thiagarajan	Chennai	Gold
to	Antalya,	Parag Agarwal	Mumbai	Gold
July 6, 2001	Turkey	Naresh Satyan	Bangalore	Silver
		S. Vijaykumar	Bangalore	Silver

2001

Delegation Leader : Prof. Vijay A. Singh (IIT, Kanpur) **Pedagogical Leader** : Prof. D. A. Desai (D. G. Ruparel College, Mumbai)

		2002		
Dates	Place	Student's Name	Place	Medals
		Ravishankar Sundararamam	Mumbai	Gold
July 21, 2002		Shantanu Bharadwaj	Mathura	Silver
to	Bandung,	Hirakendu Das	Hyderabad	Silver
July 30, 2002	Indonesia	B. Sundeep	Bangalore	Silver
		Kushal Mukherjee	Bangalore	Silver

Delegation Leader : Prof. Arvind Kumar (HBCSE, Mumbai) **Pedagogical Leader** : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi)

2003

Dates	Place	Student's Name	Place	Medals
		Yashodhan Kanoria	Mumbai	Gold
August 3, 2003		Shaleen Harlalka	Udaypur	Gold
to	Taipei,	Shashank Dwivedi	Bhilai	Bronze
August 11, 2003	Taiwan	Alekh Agarwal	Bhopal	Honourable Mention
		Divjyot Sethi	Delhi	Honourable Mention

Delegation Leader : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi) **Pedagogical Leader** : Dr. S. C. Samanta (Midnapore College, Midnapore)

		2004		
Dates	Place	Student's Name	Place	Medals
		Shubham Mittal	New Delhi	Gold
July 15, 2004		Ajit Kumar Nema	Bangalore	Silver
to	Pohang,	Kartik Mohta	Nagpur	Silver
July 23, 2004	South Korea	Avin Mittal	Agra	Bronze
		Ankur Goel	Panchkula	Bronze

Delegation Leader : Prof. Dipan Ghosh (IIT, Mumbai) **Pedagogical Leader** : Dr. Rajesh Khaparde (HBCSE, Mumbai)

Dates	Place	Student's Name	Place	Medals
		Piyush Srivastav	Allahabad	Gold
July 3, 2005		Sameer Madan	Panchkula	Gold
to	Salamanca,	Tejaswi Venumadhavan Nerella	Hyderabad	Silver
July 12, 2005	Spain	Hema Chandra Prakash Movva	Hyderabad	Silver
		Arjun Radhakrishna	Bangalore	Bronze

2005

Delegation Leader : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi) **Pedagogical Leader** : Dr. Rajesh Khaparde (HBCSE, Mumbai) **Scientific Observer** : Dr. Bhupati Chakravarti (City College, Kolkata)

		2006		
Dates	Place	Student's Name	Place	Medals
		Mehul Tikekar	Mumbai	Gold
July 8, 2006		Raghu Mahajan	Chandigarh	Gold
to	Singapore	Harish Ravi	Bangalore	Bronze
July 17, 2006		Divyanshu Jha	Patna	Bronze
		Neha Rambhia	Mumbai	Bronze

Delegation Leader : Dr. Charudatt Kadolkar (IIT, Guwahati)

Pedagogical Leader: Prof. B. N. Chandrika (VVSFG College for Women, Bangalore) **Scientific Observer**: Mr. Shirish Pathare (HBCSE, Mumbai)

2007					
Dates	Place	Student's Name	Place	Medals	
		Raman Sharma	Jaipur	Gold	
July 13, 2007		Rohit Singh	Dehradun	Gold	
to	Isfahan,	Pratyush Pandey	Jaipur	Silver	
July 22, 2007	Iran	Harsh Pareek	Mumbai	Silver	
		Vivek Lohani	Almora	Honourable Mention	

Delegation Leader : Dr. Charudatt Kadolkar (IIT, Guwahati) **Pedagogical Leader** : Prof. Vijay A. Singh (HBCSE, Mumbai)

		2008		
Dates	Place	Student's Name	Place	Medals
		Garvit Juniwal	Jaipur	Gold
July 20, 2008		Kunal Yogen Shah	Mumbai	Gold
to	Hanoi,	Nishant Totla	Aurangabad	Gold
July 29, 2008	Vietnam	Shitikanth	Patna	Gold
		Saurabh Goyal	Kota	Silver

Delegation Leader : Dr. Pramendra R. Singh (Jagdam College, J. P. University, Chhapra) **Pedagogical Leader** : Dr. Vishwajeet Kulkarni (Smt. Parvatibai Chowgule College, Goa) **Scientific Observer** : Dr. Charudatt Kadolkar (IIT, Guwahati)

Dates	Place	Student's Name	Place	Medals
		Gopi Sivakanth	Yeleswaram	Gold
July 12, 2009		Nitin Jain	Faridabad	Gold
to	Merida Yucatan,	Priyank Pradeep Parikh	Mumbai	Gold
July 19, 2009	Mexico	Shubham Tulsiani	Jodhpur	Gold
		Vinit Atal	Pune	Silver

 $\mathbf{2009}$

Delegation Leaders : (1) Prof. H. C. Pradhan (HBCSE, Mumbai)

: (2) Dr. Pramendra R. Singh (Jagdam College, J. P. University, Chhapra) Scientific Observer : Shri A. M. Shaker (K. J. Somaiya College, Mumbai)

Like the Sports Olympics, the Olympiads are individual events and there is no official ranking of nations by the International Olympiad Committee. Our ranking though unofficial is based on aggregate national scores.

Year	Countries Participated	Rank
1998	56	10
1999	62	10
2000	64	3
2001	65	4
2002	67	7
2003	54	8
2004	71	9
2005	77	8
2006	89	8
2007	70	6
2008*	82	3
2009*	76	3

 \ast 2008 and 2009 Physics Olympiad team performances (4 Golds and 1 Silver) represent best efforts by Indian team in any of the international olympiads (Astronomy, Mathematics, Physics, Chemistry and Biology) India has participated so far.

