

Common Mistakes in INMO 2021

1 Problem 1

- Claiming that the answer is 3, without constructing a valid solution for $r = 3$.
- Assuming some extra constraint(s) which are not stated by the problem. For eg. assuming that m_i, n_i are non-negative, consecutive, monotonic or have some special form etc.
- Claiming without proof that m_i are monotonic iff n_i are also monotonic.

2 Problem 2

- Assuming that a common root exists for the two equations.
- Ignoring negative sign of root while dividing both sides of an inequality by this root, which should have changed the sign of the inequality. Or applying AM-GM to these negative roots.
- Claiming $(a, b) = (0, -n^6)$ is an answer.
- Incorrectly applying an irreducibility criterion which only works if the constant term is a prime, to claim that $||a| - |b|| \leq 1$, or using a wrong statement of Perron's Irreducibility Criterion.

3 Problem 3

- Showing that one can draw the midpoint of the third side of a triangle, given the midpoints of the other two, and claiming this alone can be repeated to draw any desired midpoint. This is false: for example, if Betal chooses 337 disjoint triangles and marks the midpoints of the 1011 sides, and then above algorithm yields no new midpoints.
- Claiming that since the aforesaid construction doesn't work in general, Vikram cannot win.
- Failing to handle degenerate cases (e.g., when the construction requires the intersection of two lines, but they may happen to be parallel).
- Using the straightedge in ways other than those specified in the question (e.g., "translating" lines, drawing arbitrary parallel/perpendicular lines without justifying why those are possible with a straightedge).

4 Problem 4

- Arguing by 'probability', that the detective is unlikely to be able to get a pair of consecutive numbers in less than 50 questions.
- Showing that the detective can't assure a win in less than 50 moves for one specific strategy, arguing that the proposed strategy is the 'best'.
- Arguing by 'information', that the detective doesn't learn anything new from finding that two cards don't form a consecutive pair.
- Assuming that in the standard strategy for getting the detective to win after asking 51 questions, the detective knows the number on the fixed card.
- Assuming that the deck of cards has 4 suits with each containing an ace, a jack, a queen and a king with numbers not exceeding 10.

5 Problem 5

- Verifying a solution to some trigonometric equation without checking the uniqueness.
- Not doing complicated simplifications and directly stating how things would be.

6 Problem 6

- Assuming f is a polynomial.
- Assuming that if 2 polynomials share the same set of real roots, then they are a constant multiple of each other.
- Missing one of the solutions. ($f(P) = +P$ or $f(P) = -P$).
- Not considering the multiplicity of real roots of a polynomial.