Indian Olympiad Qualifier in Astronomy (IOQA) 2020 – 2021

conducted jointly by

Homi Bhabha Centre for Science Education (HBCSE-TIFR)

and

Indian Association of Physics Teachers (IAPT)

Part II: Indian National Astronomy Olympiad (INAO)

Homi Bhabha Centre for Science Education (HBCSE-TIFR)

Question Paper

Date: 6^{th} February 2021 | Time: **10:15 – 12:15 hrs**

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐☐☐ | Maximum Marks: 80

Please Note:

- Please write your roll number in the space provided above.
- There are total 5 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Distance of the Moon from Earth	$d_☾$	$\approx$	3.844×10^{8} m
Astronomical Unit	$a_\oplus$	$\approx$	1.496×10^{11} m
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

1. A Newtonian reflector type of telescope has a concave mirror with a 2.00 m radius of curvature as its primary mirror. It is fitted with camera at primary focus with an achromatic camera lens of 4.00 cm focal length.
   (a) (2 marks) What is the angular magnification of this system?
   (b) (3 marks) We observe a 25 000 km diameter sunspot with this system. What will be the angular size of the sunspot in the image?
   (c) (7 marks) The camera lens is now removed and an image detector is placed in such a way that a well-focused sunspot image due to primary mirror could be observed with yellow light (wavelength 550 nm). We wish to observe the same region now in green light (wavelength 465 nm). For this, we introduce a green filter, which blocks all other wavelengths except green light, in front of the detector. If the thickness of this plane parallel glass plate (refractive index 1.53) is $t = 2.887$ mm, how much will be the change in the position of the image?

Solution:

(a) As the radius of curvature is 2.00 m, hence the focal length is 1.00 m.
Angular magnification m is given by:

$$m = -\frac{F_{\text{objective}}}{F_{\text{eyepiece}}}$$
$$m = \frac{-100}{4}$$
$$\boxed{m = -25}$$

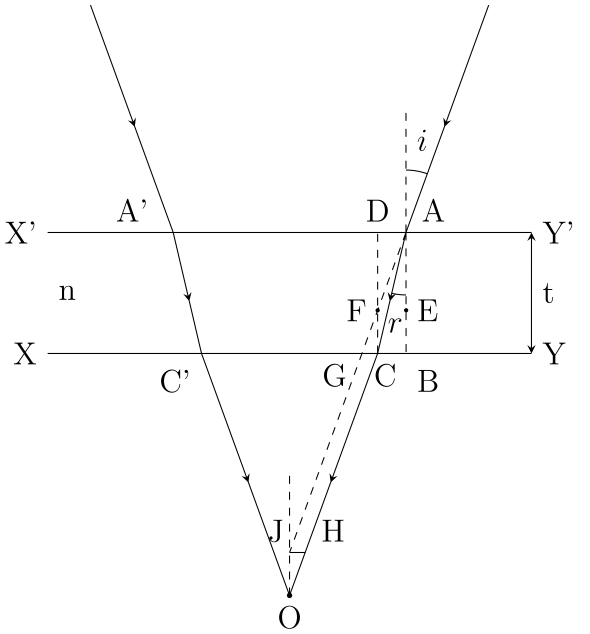
(b) Angular size of 25 000 km sunspot at the Earth:

$$\theta = \frac{\text{Diameter of Sunspot}}{\text{Distance from the Earth}}$$
$$\theta = \frac{25\,000}{1.496 \times 10^8} = 1.67 \times 10^{-4}\,\text{rad}$$
$$\theta = 34.5''$$

Thus, in the telescopic image the angle will be:

$$\theta_t = m\theta = 34.5'' \times 25$$
$$\boxed{\theta = 14'23''}$$

(c) Let, i and r be the angle of incidence and angle of refraction respectively. Let x be the change in focal length.



From the figure, it is clear that $x = JO$,

$$\begin{aligned}
\text{Now}, \angle JOH &= \angle i \\
\therefore \tan i &= \frac{JH}{x} \\
\text{but }, JH &= GC \\
\text{So }, \tan i &= \frac{GC}{x} \\
\text{In } \triangle AGB, & \\
\tan i &= \frac{GB}{AB} = \frac{GC + CB}{AB} \\
\tan r &= \frac{CB}{AB} \\
\tan i &= \frac{x \tan i}{t} + \tan r \\
\left(1 - \frac{x}{t}\right) \tan i &= \tan r
\end{aligned}$$

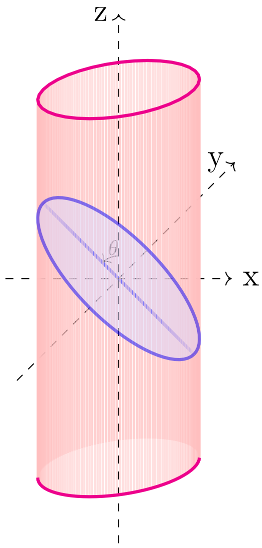
As the angles are small

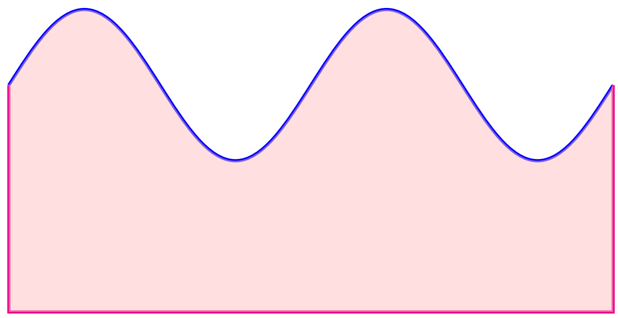
$$\begin{aligned}
\text{refractive index } n &= \frac{\sin i}{\sin r} \approx \frac{\tan i}{\tan r} \\
n &= \frac{t}{t - x} \\
nt - nx &= t \\
\frac{(n-1)t}{n} &= x \\
\therefore x &= \frac{0.53 \times 2.887}{1.53} \\
&\boxed{x = 1.0\,\text{mm}}
\end{aligned}$$

2. A rectangular sheet of paper was rolled to form a cylinder, with exactly two layers of sheet along the curved surface. This cylinder is cut such that the plane of the cut makes an angle of 45° with the axis of cylinder. The paper was then unrolled and spread on a flat table.
   (a) (2 marks) Draw a figure to show how the unrolled paper would appear.
   (b) (4 marks) Justify your answer with appropriate mathematical arguments.

Solution:

(a)





(b) Imagine cutting the cylinder perpendicular to z-axis, in which case, top edge of the paper (blue) will still be flat.
If we give a cut at angle θ to the z-axis, the bottom-most point and top-most point would shift vertically by $r \tan\theta$ from x-y plane.
In fact every point will shift vertically by $\pm x \tan\theta$.
Let ϕ be the azimuth in the x-y plane.
As paper is rolled to form two layers, length of the paper is twice the circumference of the cylinder, i.e. $l = 4\pi r$

$$\therefore r = \frac{l}{4\pi}$$
$$x = r\cos\phi$$

$\therefore$ vertical displacement is

$$z = \frac{l}{4\pi}\cos\phi\tan\theta$$
$$\because \theta = 45^\circ$$
$$\boxed{\therefore z = \frac{l}{4\pi}\cos\phi}$$

Thus, it is exactly a sine / cosine curve.

Note: If the radius of the cylinder is very large as compared to the height, then in this special case a different solution is possible, which will also get credit.

3. Betelgeuse, a red supergiant star, in the constellation of Orion, is known as an irregular variable star. It's magnitude varies between +0.3 to +1.0 from time to time. However, last year the astronomers were surprised to observe an unexpected dimming of Betelgeuse. We may assume this event started from 12 October 2019. Given below is a plot of the observed magnitude vs time (light-curve) of Betelgeuse.

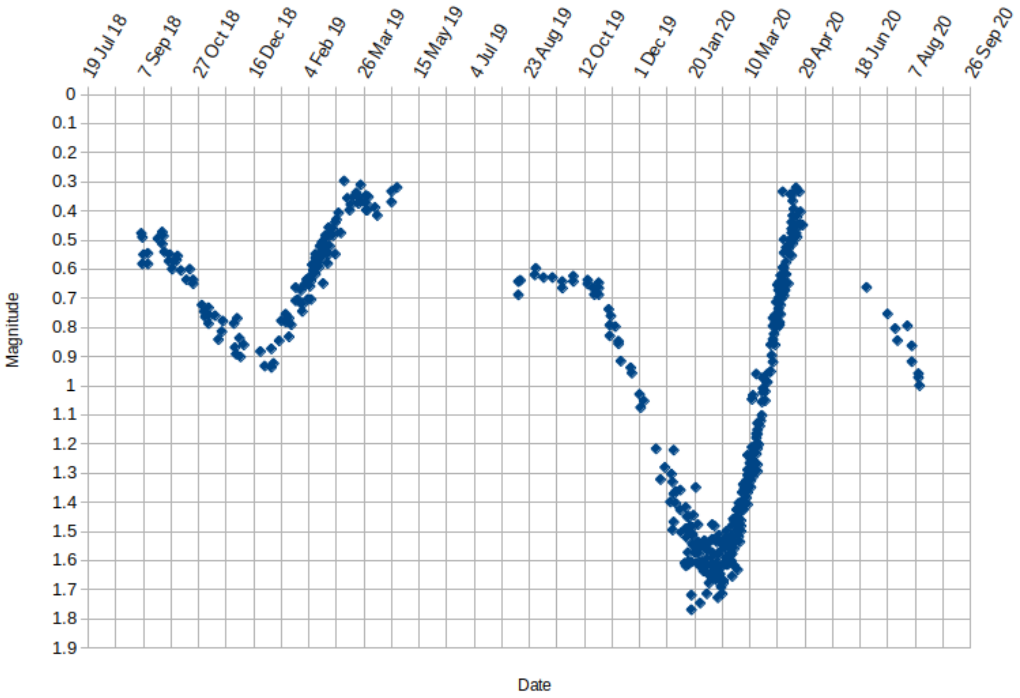


Figure 1: The V band magnitudes are observations from the AAVSO International Database (https://www.aavso.org)

Notes:

- The relation between magnitude of the star and light flux received from it is given by :

$$m_1 - m_2 = -2.5\ log_{10}\left(\frac{F_1}{F_2}\right)$$

  where m_1 and m_2 are magnitudes measured in two different observations and F_1 and F_2 are corresponding light fluxes.
- Mass of Betelgeuse: $M_B = 2.1 \times 10^{31}$ kg
- Distance of Betelgeuse from the Earth: $d_B = 200\,\text{pc}$
- Typical radius of Betelgeuse: $R_1 = 6.17 \times 10^{11}$ m

(a) (8 marks) One of the proposed model for this dimming was that the whole star suddenly started expanding and hence cooled down. Let us assume that the star is still acting as perfect black body at each stage of expansion (and subsequent contraction). By other measurements, we know that the star's effective temperature at the start of expansion was $T_1 = 3500\,\text{K}$ and the effective temperature at the most expanded state is $T_2 = 2625\,\text{K}$. Find the average velocity of the expansion of gas.

(b) (9 marks) Some other astronomers proposed that the said dimming is caused due to the transit of a giant exoplanet with radius 'r' orbiting Betelgeuse. Argue if such a scenario is possible for an edge-on circular orbit of the exoplanet with orbital radius 'a'.

(c) (7 marks) A popular model to explain this dimming states that this event started with a large plume of hot material getting ejected from the star's surface. This material cooled down after ejection and became opaque to block light from a part of the star. As this dense cloud expanded, it kept blocking more and more part of the star dimming it further. However, as this expansion lowered the density of the cloud, the cloud's opaqueness started reducing after a few weeks and the star started brightening again.

Here we will consider a simpler version of this model. We assume that this material ejection happened in a narrow cone in very short timescale from a single point on the non-rotating stellar surface. The total mass of the ejected material was approximately equal to mass of the earth and the axis of the cone was exactly along our line of sight. Let us assume that at each instant during the expansion the density of material is constant throughout the cone and the vertex of the cone is still attached to the stellar surface.

We assume that the star starts brightening again when the average density inside the cone falls to $5 \times 10^{-14}\,\mathrm{kg\,m^{-3}}$. Find the time averaged velocity of particles, which form the front of the expanding cone.

Solution:

(a) When the star is in the most expanded state the radius is large, temperature is low and the magnitude is minimum. On the other hand, when it is contracted the radius is small, temperature is high and the magnitude is maximum.
Here we will consider only the period starting from 12 October 2019 when the magnitude is maximum till the magnitude is minimum which around 14 February 2020.
The magnitude is maximum on 12 Oct 2019 $m_x = 0.65$.
The magnitude is minimum on 14 Feb 2020 $m_c = 1.60$.
This gives the time period from the most contracted state to most expanded state to be $t_{cx} = 125\,\mathrm{days} = 1.08 \times 10^7\,\mathrm{s}$.

At the start of expansion,
radius:$R_1 = 6.17 \times 10^{11}\,\mathrm{m}$, temperature: $T_1 = 3500\,\mathrm{K}$ and magnitude: $m_1 = 0.65$
For the most expanded state,
radius: R_2, temperature: $T_2 = 2625\,\mathrm{K}$ and magnitude: $m_2 = 1.60$

$$
\begin{aligned}
m_1 - m_2 &= -2.5\ log_{10}\left(\frac{F_1}{F_2}\right) \\
\frac{F_1}{F_2} &= 10^{-0.4(m_1 - m_2)} \\
\frac{F_1}{F_2} &= 10^{-0.4(0.65-1.60)} \\
\frac{F_1}{F_2} &= 2.399 \\
L &= 4\pi R^2 \sigma T^4 \\
F &= \frac{4\pi R^2 \sigma T^4}{4\pi D^2}
\end{aligned}
$$

Here D is the distance to Betelgeuse.

$$\frac{F_1}{F_2} = \frac{R_1^2 \times T_1^4}{R_2^2 \times T_2^4}$$
$$\frac{R_1}{R_2} = \sqrt{\frac{F_1}{F_2}} \times \frac{T_2^2}{T_1^2}$$
$$\frac{R_1}{R_2} = \sqrt{2.399} \times \frac{2625^2}{3500^2}$$
$$\frac{R_1}{R_2} = 0.871$$
$$R_2 = \frac{6.17 \times 10^{11}}{0.871}$$
$$\boxed{R_2 = 7.08 \times 10^{11}\,\mathrm{m}}$$

Therefore, change in the Radius

$$R_2 - R_1 = 9.1 \times 10^{10}\,\mathrm{m}$$

and the velocity of expansion v_{exp} is:

$$v_{\mathrm{exp}} = \frac{\text{Change in Radius}}{t_{cx}}$$
$$v_{\mathrm{exp}} = \frac{9.1 \times 10^{10}}{1.08 \times 10^7}$$
$$\boxed{v_{\mathrm{exp}} = 8.4\,\mathrm{km\,s^{-1}}}$$

(b) Transit time from first contact to second contact $t = 1.08 \times 10^7\,\mathrm{s}$ [from part a].
Flux ratio = 2.399 [from part a]
Let P be the period of the orbit and a be the orbital radius then,

$$P^2 = \frac{4\pi^2}{GM_{\mathrm{B}}} a^3$$

The angle travelled in time t be θ;

$$\therefore \frac{\theta}{2\pi} = \frac{t}{P} \Rightarrow \theta = \frac{2\pi t}{P}$$

Distance travelled in apparent view $= r + R_{\mathrm{B}} = a \sin\theta$

$$\theta = t\sqrt{\frac{GM_{\mathrm{B}}}{a^3}} = \sin^{-1}(r + R_{\mathrm{B}})$$

If θ is small,

$$
\begin{aligned}
t\sqrt{\frac{GM_\mathrm{B}}{a^3}} &= r + R_\mathrm{B} \\
\therefore 4.04 \times 10^{17} &= \sqrt{a^3}(r + R_\mathrm{B}) \\
\sqrt{a^3} &= \frac{4.04 \times 10^{17}}{(r + R_\mathrm{B})} < \frac{4.04 \times 10^{17}}{R_\mathrm{B}} = 6.54 \times 10^5 \\
\therefore a &< \left(6.54 \times 10^5\right)^{2/3} \\
a &< 7541\,\mathrm{km} \\
\therefore a &\ll R_\mathrm{B}
\end{aligned}
$$

This is physically impossible.

(c) Total mass of gas ejected : $5.972 \times 10^{24}\,\mathrm{kg}$
Hence, volume V of the cone, when the star starts brightening again, is:

$$V = \frac{\text{Mass}}{\text{Density}} = \frac{5.972 \times 10^{24}}{5 \times 10^{-14}} \approx 1.2 \times 10^{38}\,\mathrm{m}^3$$

Let r be base of the cone and h be the height. Now as per the first part, flux ratio from the start of emission to attainment of critical density is 2.399. As the flux reduction is happening due to circular disk of base obscuring part of the star,

$$
\begin{aligned}
\frac{F_1}{F_2} &= \frac{R_B^2}{(R_B^2 - r^2)} \\
\therefore r^2 &= \frac{R_B^2(F_1/F_2 - 1)}{F_1/F_2}
\end{aligned}
$$

The volume of the cone is given by

$$
\begin{aligned}
V &= \pi \times r^2 \times \frac{h}{3} \\
&= \frac{\pi}{3} \times \left(\frac{R_B^2(F_1/F_2 - 1)}{F_1/F_2}\right) \times h \\
1.2 \times 10^{38} &= \frac{\pi \times (6.17 \times 10^{11})^2 \times (2.399 - 1)}{2.399 \times 3} \times h \\
\therefore h = vt &= \frac{1.2 \times 10^{38}}{2.32 \times 10^{23}} = 5.1 \times 10^{14}\,\mathrm{m} \\
\therefore v &= \frac{5.1 \times 10^{14}}{1.08 \times 10^7} \\
&\approx 4.38 \times 10^7\,\mathrm{m\,s^{-1}} \\
&\boxed{v \approx 48\,000\,\mathrm{km\,s^{-1}}}
\end{aligned}
$$

4. (20 marks) A space agency would like to put an artificial satellite in a highly elliptical orbit around the Earth in the ecliptic plane (the plane of the Earth's orbit around the Sun). What can be the maximum eccentricity (e_max) for such an orbit? Also provide the perigee (r_min) and apogee (r_max)

distances (in km) from the centre of the Earth.

Solution:

The eccentricity e of an elliptical orbit is

$$e = \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}} = 1 - \frac{r_{\min}}{a} = \frac{r_{\max}}{a} - 1$$

where $r_{\max}$ and $r_{\min}$ are, respectively, the maximum and minimum distances from the focus (apogee and perigee, for an object around the Earth, the focus being the Earth's centre), and a is the semi-major axis of the ellipse.

For maximum eccentricity, we need to minimise perigee and maximise apogee distances.

In the present case, the closest perigee that is possible is a point just above the Earth's atmosphere, i.e., where the atmospheric drag is sufficiently small to allow a stable orbit.

We take a typical height of $h_{\text{atm}} \approx 100\,\text{km}$ for the Earth's atmosphere, and thus

$$r_{\min} = R_{\oplus} + h_{\text{atm}} \approx 6.471 \times 10^{6}\,\text{m}$$

(Remark: Any value $100\,\text{km} \leq h_{\text{atm}} \leq 5000\,\text{km}$ will be accepted.)

For the maximum apogee distance, one has to consider the maximum distance where the gravitational influence of the Earth would still dominate that of the Sun or the Moon so that a stable orbit around the Earth is maintained.

Approximations: In the following, we assume circular orbits of the Earth and the Moon around the Sun and the Earth, respectively. Further, we neglect any small perturbations that may destabilize the orbit of the satellite even when it is dominated by the gravitational attraction of the nearest large body (Earth/Moon).

Considering only the Sun-Earth system, in the limiting case, the apogee would be a point on the line joining the Earth to the Sun that revolves along with the Earth (around the Sun). In other words, in the limit, the centripetal acceleration of the satellite towards the Earth would vanish, and the only remaining acceleration would be the orbital acceleration around the Sun, which is still necessary for the satellite to maintain its position with respect to the Earth as the latter revolves around the Sun.

If $d_{\odot}(\equiv a_{\oplus})$ is the distance of the Sun from the Earth, and d_1 is the maximum distance from the Earth where an object would still orbit the Earth (and not the Sun directly),

$$\frac{GM_{\odot}}{(d_{\odot} - d_1)^2} - \frac{GM_{\oplus}}{d_1^2} = \omega_{\oplus}^2 (d_{\odot} - d_1)$$

where $\omega_{\oplus}$ is angular velocity of Earth around Sun

$$\omega_{\oplus}^2 = \frac{GM_{\odot}}{d_{\odot}^3}$$

Alternatively, the equation of motion at the limiting position in the satellite's non-inertial frame of reference is

$$\frac{GmM_{\odot}}{(d_{\odot} - d_1)^2} - \frac{GmM_{\oplus}}{d_1^2} - m\omega_{\oplus}^2 (d_{\odot} - d_1) = 0$$

where m is the mass of the satellite and the last term is the centrifugal force in the frame revolving around the Sun.

Either equation yields

$$\frac{M_\odot}{(d_\odot - d_1)^2} - \frac{M_\odot}{d_\odot^3}(d_\odot - d_1) - \frac{M_\oplus}{d_1^2} = 0$$

Dividing by $M_\odot/d_\odot^2$,

$$\left(1 - \frac{d_1}{d_\odot}\right)^{-2} - \left(1 - \frac{d_1}{d_\odot}\right) - \frac{M_\oplus/M_\odot}{(d_1/d_\odot)^2} = 0$$

We expect the ratio $d_1/d_\odot \ll 1$. Retaining only first order terms in $d_1/d_\odot$, we get

$$\left(1 + 2\frac{d_1}{d_\odot}\right) - \left(1 - \frac{d_1}{d_\odot}\right) = \frac{M_\oplus/M_\odot}{(d_1/d_\odot)^2}$$

$$\implies d_1 = \sqrt[3]{\frac{M_\oplus}{3M_\odot}}\, d_\odot \approx 1.496 \times 10^9\,\mathrm{m}$$

The distance d_1 is the radius of the so-called Hill sphere of the Earth.

For the Moon, we can similarly calculate d_2', which is the distance from the Moon to be within the Moon's gravitational sphere of influence. This can be done by replacing $M_\oplus \to M_☾$, $M_\odot \to M_\oplus$ and $d_\odot \to d_☾$ in the above formula. We get,

$$d_2' = \sqrt[3]{\frac{M_☾}{3M_\oplus}}\, d_☾ \approx 0.616 \times 10^8\,\mathrm{m}$$

Thus, the maximum distance d_2 from the Earth towards the Moon where the gravitational influence of the Earth would dominate is given by

$$d_2 = d_☾ - d_2' = 3.228 \times 10^8\,\mathrm{m}$$

We find that $d_2 < d_1$. So if the apogee of the satellite is at a distance less than d_2, it would never be captured by the Moon (or by the Sun, of course).
Thus the maximum eccentricity in this case would be

$$e_{\max} = \frac{d_2 - r_{\min}}{d_2 + r_{\min}} = \frac{3.228 \times 10^8 - 6.471 \times 10^6}{3.228 \times 10^8 + 6.471 \times 10^6} = 0.961$$

However, it is also possible for the satellite to be in a larger orbit in such a manner that it never enters the Hill sphere of the Moon. For this to happen over a long time, the satellite - in a pro-grade orbit - must have the same period as the Moon and also be out of phase with the Moon at perigee and apogee. For the period to be exactly the same as that of the Moon, the semi-major axis of the satellite's orbit must be the same as that of the lunar orbit, i.e., the (average) Earth-Moon distance ($d_☾$).
Therefore, the eccentricity in this case will be,

$$e_{\max} = 1 - \frac{r_{\min}}{d_☾}$$

$$= 1 - \frac{6.471 \times 10^6\,\mathrm{m}}{3.844 \times 10^8\,\mathrm{m}}$$

$$\boxed{e_{\max} = 0.983}$$

In principle, a student may wonder about other exactly out of phase resonant orbits. An orbit with period exactly twice that of revolution period of the moon will have higher eccentricity ($e' = 0.989$) and satellite in this orbit won't enter the moon's Hill sphere at any phase. We cannot aim for 1:3 or higher resonance as the apogee will be beyond the Hill sphere.

However, proving that 1:2 resonance actually keeps satellite outside Hill sphere of the Moon is beyond the scope of this examination.

5. (18 marks) Five friends from various cities of India observed the Sun and made the following statements. For their observations of shadows they all used a metre stick placed vertically on a flat ground.

    1. I observed sunrise at 04:56 on 12$^{\text{th}}$ June.
    2. I observed sunrise at 05:24 on 12$^{\text{th}}$ June, which was the second earliest out of the five cities on that day.
    3. I observed sunset at 16:55 on 24$^{\text{th}}$ December.
    4. I observed sunset at 17:35 on 24$^{\text{th}}$ December, which was the third earliest out of the five cities on that day.
    5. I observed sunset at 18:50 on 1$^{\text{st}}$ September, which was the last sunset on that day.
    6. At local noon (i.e. noon as per each local time) on 21$^{\text{st}}$ June, shadow at my location was the longest amongst all.
    7. The shortest shadow of the year at my location was observed on 21$^{\text{st}}$ June.
    8. The shortest shadow of the year at my location was observed on 5$^{\text{th}}$ June.
    9. The shortest shadow of the year at my location was observed on 26$^{\text{th}}$ May.
    10. The shortest shadow of the year at my location was observed on 15$^{\text{th}}$ April.
    11. On 1$^{\text{st}}$ July, I had a longer day as compared to other observers.
    12. On 1$^{\text{st}}$ February, I had a longer day as compared to other observers.

    Below are the locations of our observers along with the coordinates of their cities;

Observer	**Location**	**Coordinates**
Kamal	Kolkata	22.57° N, 88.36° E
Naeem	Nagpur	21.15° N, 79.09° E
Chandrika	Chandigarh	30.73° N, 76.78° E
Kate	Kochi	9.93° N, 76.27° E
Mayank	Mumbai	19.08° N, 72.88° E

    Assume that all observers have their watches synchronised to correct Indian Standard Time. Find out, for each statement, which statement was made by which observer?

    Note: You don't have to give reasons. Only a table with observer name and statement numbers is enough. Each correct pair gives you 1.5 marks. However, for each wrong pair, you will lose 0.5 mark.

Solution:

As Kolkata is significantly to the east, on all days, it will be the first [1] city to experience sunrise and sunset. Let us calculate time difference, caused only due to longitude difference, between sunrise / sunset at other cities as compared to Kolkata.

Observer	**Location**	**Coordinates**	$\Delta\lambda$	Time diff. due to λ
Kamal	Kolkata	22.57° N, 88.36° E	0.0°	00:00
Naeem	Nagpur	21.15° N, 79.09° E	9.3°	00:37
Chandrika	Chandigarh	30.73° N, 76.78° E	11.6°	00:46
Kate	Kochi	9.93° N, 76.27° E	12.1°	00:48
Mayank	Mumbai	19.08° N, 72.88° E	15.5°	01:02

In other words, if latitudes were not a factor, we would expect Nagpur sunrise to happen 37 minutes after Kolkata, Chandigarh 46 minutes after Kolkata and so on. Same would be true for the sunset.

But, we know that in the northern places, days are significantly longer in the summer and significantly shorter in the winter. Thus, we expect some latitude dependence.

One should also recall the Sun is directly in front of the equator on only the equinox days. On the summer solstice day the sun is in front of tropic of cancer ($\phi = +23.44°$) and on the winter solstice day it is in front of the tropic of Capricorn ($\phi = -23.44°$).

- Let's consider only sunrise statements (no. 1 and 2). Both the statements are about sunrise on 12th June, which is very close to the summer solstice day. As 05:24 is the second earliest sunrise, it is obvious that 04:56 is the earliest and hence it must be a statement by Kamal.
- If Kolkata sunrise was at 04:56, as per our table, we would expect Nagpur sunrise at 05:33. But there is a sunrise at 05:24, which may be caused due to northern latitude of the observer. Thus, statement 2 must be by Chandrika.
- Consider the Sunset statements 3 and 4, related to observations near the winter solstice day. As 17:35 is the third earliest of the 5 sunsets, it must be of Nagpur. Kolkata is significantly more eastwards and Chandigarh is significantly more northwards. So they will have earlier sunsets. On the other hand, Kochi and Mumbai are both more towards west and also towards south as compared to Nagpur. Thus 4 belongs to Naeem.
- Now sunset in 3 was 40 minutes prior to the Nagpur sunset. For Kolkata we expect the sunset to be 37 minutes earlier due to longitude difference and since Kolkata's latitude is 1° more than that of Nagpur, we expect the day to be slightly shorter. So 40 minute difference matches with Kolkata.
- In statement 5 we are close to the equinox day. Thus day lengths would not differ significantly at different latitudes and the order of the sunsets will be mostly be determined by the order of longitudes. Thus, last sunset will be observed by Mayank.
- On 21st June, the sun will be in front of the tropic of cancer ($\phi = +23.44°$) and altitude of the Sun at local noon will depend on distance (latitude difference) of these cities from the tropic of cancer. Thus, statement 6 is made by Kate.

- Next consider statements 7 to 10. These clearly belong to four different cities. All cities to the south of the tropic of cancer would experience 'zero shadow' at local noon on one of the days between vernal equinox and summer solstice. The zero shadow day occurs in the increasing order of the latitudes with faster change between 21 Mar - 15 May (approx $0°$ to $19°$) and much slower change between 15 May - 21 Jun ($19°$ to $23.44°$). So statement 10 is made by Kate, 9 is made by Naeem and 8 is made by Kamal. For all places to the north of the tropic of cancer, the shortest shadow will be seen on the summer solstice day. Thus, statement 7 will be made by Chandrika.

- Lastly, for statements 11 and statement 12, we note that Chandigarh (highest latitude) will have longer day in the summer and Kochi (lowest latitude) will have the longest day in the winter.

Thus, the final table is,

Observer	**Location**	**Statement No.**
Kamal	Kolkata	1, 3, 8
Naeem	Nagpur	4, 9
Chandrika	Chandigarh	2, 7, 11
Kate	Kochi	6, 10, 12
Mayank	Mumbai	5

Notes:

- For statements 1 to 5, the correction due to equation of time can be safely ignored as the same is almost zero on the given dates.

- The sunrise / sunset times / altitudes / dates of zero shadow day may be exactly calculated using the principles of spherical geometry. That is also a valid justification but not a necessary one.