Indian Olympiad Qualifier in Astronomy (IOQA) 2020 – 2021

conducted jointly by

Homi Bhabha Centre for Science Education (HBCSE-TIFR)

and

Indian Association of Physics Teachers (IAPT)

Part II: Indian National Astronomy Olympiad (INAO)

Homi Bhabha Centre for Science Education (HBCSE-TIFR)

Question Paper

Date: 6th February 2021 Time: **10:15 – 12:15 hrs**

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐☐☐ Maximum Marks: 80

Please Note:

- Please write your roll number in the space provided above.
- There are total 5 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Distance of the Moon from Earth	$d_☾$	$\approx$	3.844×10^{8} m
Astronomical Unit	$a_\oplus$	$\approx$	1.496×10^{11} m
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

1. A Newtonian reflector type of telescope has a concave mirror with a 2.00 m radius of curvature as its primary mirror. It is fitted with camera at primary focus with an achromatic camera lens of 4.00 cm focal length.
   (a) (2 marks) What is the angular magnification of this system?
   (b) (3 marks) We observe a 25 000 km diameter sunspot with this system. What will be the angular size of the sunspot in the image?
   (c) (7 marks) The camera lens is now removed and an image detector is placed in such a way that a well-focused sunspot image due to primary mirror could be observed with yellow light (wavelength 550 nm). We wish to observe the same region now in green light (wavelength 465 nm). For this, we introduce a green filter, which blocks all other wavelengths except green light, in front of the detector. If the thickness of this plane parallel glass plate (refractive index 1.53) is $t = 2.887$ mm, how much will be the change in the position of the image?

2. A rectangular sheet of paper was rolled to form a cylinder, with exactly two layers of sheet along the curved surface. This cylinder is cut such that the plane of the cut makes an angle of 45° with the axis of cylinder. The paper was then unrolled and spread on a flat table.
   (a) (2 marks) Draw a figure to show how the unrolled paper would appear.
   (b) (4 marks) Justify your answer with appropriate mathematical arguments.

3. Betelgeuse, a red supergiant star, in the constellation of Orion, is known as an irregular variable star. It's magnitude varies between +0.3 to +1.0 from time to time. However, last year the astronomers were surprised to observe an unexpected dimming of Betelgeuse. We may assume this event started from 12 October 2019. Given below is a plot of the observed magnitude vs time (light-curve) of Betelgeuse.

   Notes:

   - The relation between magnitude of the star and light flux received from it is given by :

     $$m_1 - m_2 = -2.5\ log_{10}\left(\frac{F_1}{F_2}\right)$$

     where m_1 and m_2 are magnitudes measured in two different observations and F_1 and F_2 are corresponding light fluxes.
   - Mass of Betelgeuse: $M_B = 2.1 \times 10^{31}$ kg
   - Distance of Betelgeuse from the Earth: $d_B = 200$ pc
   - Typical radius of Betelgeuse: $R_1 = 6.17 \times 10^{11}$ m

   (a) (8 marks) One of the proposed model for this dimming was that the whole star suddenly started expanding and hence cooled down. Let us assume that the star is still acting as perfect black body at each stage of expansion (and subsequent contraction). By other measurements, we know that the star's effective temperature at the start of expansion was $T_1 = 3500$ K and the effective temperature at the most expanded state is $T_2 = 2625$ K. Find the average velocity of the expansion of gas.
   (b) (9 marks) Some other astronomers proposed that the said dimming is caused due to the transit of a giant exoplanet with radius 'r' orbiting Betelgeuse. Argue if such a scenario is possible for an edge-on circular orbit of the exoplanet with orbital radius 'a'.

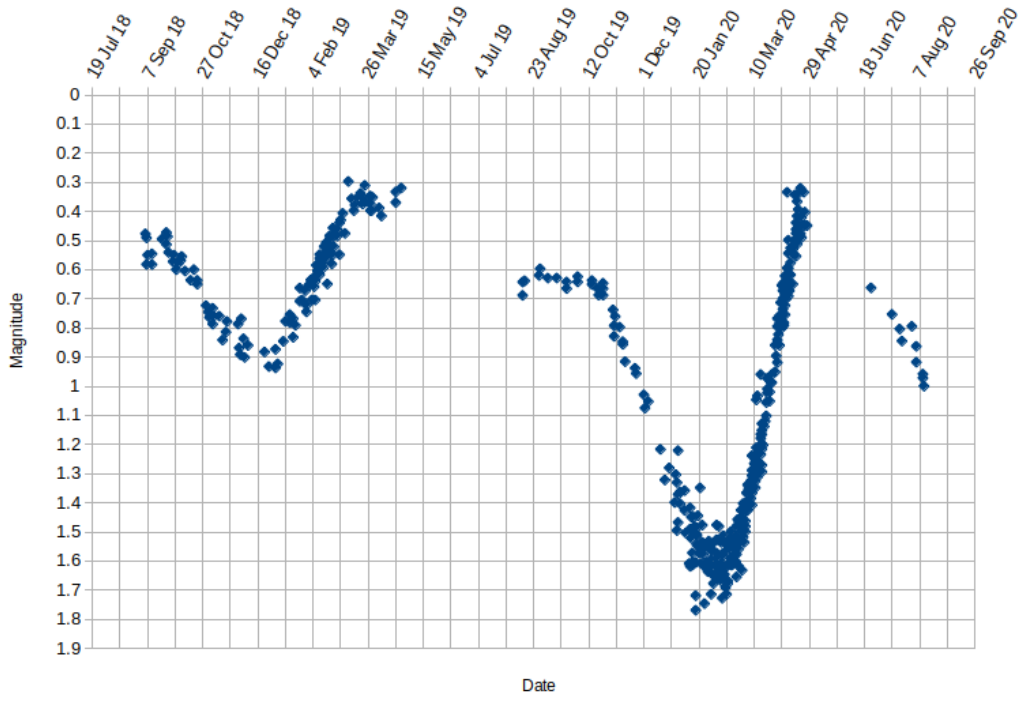

Figure 1: The V band magnitudes are observations from the AAVSO International Database (https://www.aavso.org)

(c) (7 marks) A popular model to explain this dimming states that this event started with a large plume of hot material getting ejected from the star's surface. This material cooled down after ejection and became opaque to block light from a part of the star. As this dense cloud expanded, it kept blocking more and more part of the star dimming it further. However, as this expansion lowered the density of the cloud, the cloud's opaqueness started reducing after a few weeks and the star started brightening again.

Here we will consider a simpler version of this model. We assume that this material ejection happened in a narrow cone in very short timescale from a single point on the non-rotating stellar surface. The total mass of the ejected material was approximately equal to mass of the earth and the axis of the cone was exactly along our line of sight. Let us assume that at each instant during the expansion the density of material is constant throughout the cone and the vertex of the cone is still attached to the stellar surface.

We assume that the star starts brightening again when the average density inside the cone falls to $5 \times 10^{-14}\,\mathrm{kg\,m^{-3}}$. Find the time averaged velocity of particles, which form the front of the expanding cone.

4. (20 marks) A space agency would like to put an artificial satellite in a highly elliptical orbit around the Earth in the ecliptic plane (the plane of the Earth's orbit around the Sun). What can be the maximum eccentricity ($e_{\max}$) for such an orbit? Also provide the perigee ($r_{\min}$) and apogee ($r_{\max}$) distances (in km) from the centre of the Earth.

5. (18 marks) Five friends from various cities of India observed the Sun and made the following statements. For their observations of shadows they all used a metre stick placed vertically on a flat ground.

    1. I observed sunrise at 04:56 on 12^{th} June.
    2. I observed sunrise at 05:24 on 12^{th} June, which was the second earliest out of the five cities on that day.
    3. I observed sunset at 16:55 on 24^{th} December.
    4. I observed sunset at 17:35 on 24^{th} December, which was the third earliest out of the five cities on that day.
    5. I observed sunset at 18:50 on 1^{st} September, which was the last sunset on that day.
    6. At local noon (i.e. noon as per each local time) on 21^{st} June, shadow at my location was the longest amongst all.
    7. The shortest shadow of the year at my location was observed on 21^{st} June.
    8. The shortest shadow of the year at my location was observed on 5^{th} June.
    9. The shortest shadow of the year at my location was observed on 26^{th} May.
    10. The shortest shadow of the year at my location was observed on 15^{th} April.
    11. On 1^{st} July, I had a longer day as compared to other observers.
    12. On 1^{st} February, I had a longer day as compared to other observers.

    Below are the locations of our observers along with the coordinates of their cities;

Observer	**Location**	**Coordinates**
Kamal	Kolkata	22.57° N, 88.36° E
Naeem	Nagpur	21.15° N, 79.09° E
Chandrika	Chandigarh	30.73° N, 76.78° E
Kate	Kochi	9.93° N, 76.27° E
Mayank	Mumbai	19.08° N, 72.88° E

    Assume that all observers have their watches synchronised to correct Indian Standard Time. Find out, for each statement, which statement was made by which observer?

    Note: You don't have to give reasons. Only a table with observer name and statement numbers is enough. Each correct pair gives you 1.5 marks. However, for each wrong pair, you will lose 0.5 mark.