Indian National Astronomy Olympiad – 2019

Question Paper | INAO – 2019

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ | Date: 2nd February 2019

Duration: **Three Hours** | Maximum Marks: 100

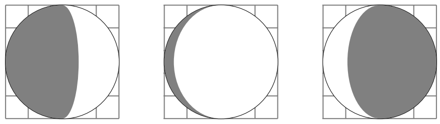
Please Note:

- Before starting, please ensure that you have received a copy of the question paper containing a cover page and 12 pages of questions.
- Please write your roll number in the space provided above.
- There are total 7 questions. Maximum marks are indicated in front of each question / sub-question.
- For all questions, the process involved in arriving at the solution is more important than the final answer. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning / assumptions / approximations.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question paper back with you.
- Please take note of following details about Orientation-Cum-Selection Camp (OCSC) in Astronomy:
  - Tentative Dates: 22nd April to 9th May 2019.
  - This camp will be held at HBCSE, Mumbai.
  - Attending the camp for the entire duration is mandatory for all participants.

Useful Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Mass of the Earth	$M_\oplus$	$\approx$	5.972×10^{24} kg
Mass of the Moon	$M_☾$	$\approx$	7.347×10^{22} kg
Radius of the Earth	$R_\oplus$	$\approx$	6.371×10^{6} m
Speed of Light	c	$\approx$	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
Radius of the Moon	R_m	$\approx$	1.737×10^{6} m
Astronomical Unit	$a_\oplus$	$\approx$	1.496×10^{11} m
Solar Constant (at Earth)	S	$\approx$	$1366\,\mathrm{W\,m^{-2}}$
Gravitational Constant	G	$\approx$	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

1. We can represent phases of the Moon diagramatically as follows:



It is known that there would be a new moon day on 28$^{\text{th}}$ February 2025. Sameer, who is writing a science fiction story, visualised various hypothetical situations for the phases of Moon during the month of March 2025. For each of the cases described below, draw the phases of Moon as it will appear on 3$^{\text{rd}}$ March, 8$^{\text{th}}$ March, 11$^{\text{th}}$ March, 15$^{\text{th}}$ March, 20$^{\text{th}}$ March and 27$^{\text{th}}$ March. High precision diagrams are not expected. Any approximate drawings (like the ones shown above) will be accepted.

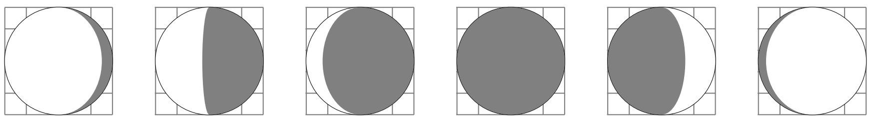
(a) (4 marks) First he thought that the lead character of his novel should be a resident of the Moon and she can see the Earth in her sky. Draw phases of Earth as seen by her.

(b) (4 marks) As he did not find first case interesting enough, he brought his leading lady back to the Earth. Next he assumed that the orbit of Moon around Earth is perpendicular (inclination=90°) to the orbit of Earth around the Sun (i.e. ecliptic plane) and the normal to the orbit is always towards the Sun. Draw phases of the Moon in this case.

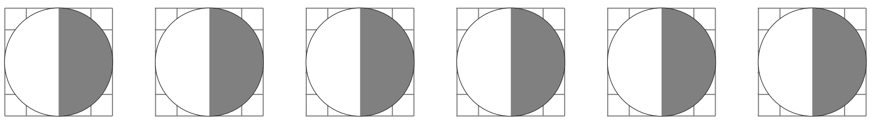
(c) (4 marks) Lastly, he assumed the orbit of the Moon is in the ecliptic plane (inclination=0°), but the shape of the Moon is cylindrical, with the diameter of the cylinder being equal to its height. The axis of the cylinder is perpendicular to the ecliptic plane. For the observer based on Earth, draw the phases of the Moon.

Solution:

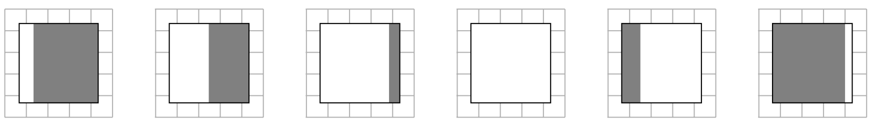
(a) By visualising the situation, we realise that $\text{phase}_{\oplus} + \text{phase}_m = \pi$



(b) In this case, Moon will always be seen as half Moon.



(c) The vertical cross section of a cylinder is a rectangle. Thus,



On the full Moon day, one may see a total lunar eclipse. But the eclipse may not last the full night. So if the full Moon day picture shows eclipse, it should be clearly mentioned. Corrections for liberation and geocentric offset of the Moon are ignored in this simplified solution.

2. Below is a table of data indicating orbital radii and masses of solar system planets and some dwarf planets. Assume all orbits to be circular around the centre of mass of the solar system (C) and all objects to be in ecliptic plane.

Object	**Mass** (kg)	**Orbital Radius** (km)		**Object**	**Mass** (kg)	**Orbital Radius** (km)
Mercury	3.301×10^{23}	5.791×10^{7}		Saturn	5.683×10^{26}	1.427×10^{9}
Venus	4.867×10^{24}	1.082×10^{8}		Uranus	8.681×10^{25}	2.871×10^{9}
Earth	5.972×10^{24}	1.496×10^{8}		Neptune	1.024×10^{26}	4.498×10^{9}
Mars	6.417×10^{23}	2.279×10^{8}		Pluto	1.309×10^{22}	5.906×10^{10}
Ceres	9.470×10^{20}	4.137×10^{8}		Eris	1.660×10^{22}	1.018×10^{11}
Jupiter	1.898×10^{27}	7.783×10^{8}		–	–	–

(a) (3 marks) Either qualitatively describe or schematically draw what should be the configuration of these bodies around the Sun such that,

(i) point C is as far as possible from the centre of the Sun (S).

(ii) points C and S are as close to each other as possible.

(b) (7 marks) Assuming all planets to be collinear, estimate the smallest possible distance between points C and S.

Solution:

(a) Centre of mass of the solar system (C) is farthest away from the centre of the Sun (S) when all objects are along one line and all planets are on the same side of the sun.

For C and S to be the closest, the ideal configuration may have the planets distributed in the ecliptic plane such that their moments along the axis perpendicular to Sun-Jupiter line will cancel each other.

Since Jupiter is the second heaviest object in our solar system and masses of all other objects are smaller as compered to the Jupiter, we intuitively expect this to happen when Jupiter is on one side of the Sun and all other objects are on the other side.

(b) Let the centre of the solar system be at the origin.

$$\begin{aligned} 0 &= M_{\odot}\vec{r_{\odot}} + M_{me}\vec{r_{me}} + M_v\vec{r_v} + M_{\oplus}\vec{r_{\oplus}} + M_{ma}\vec{r_{ma}} + M_J\vec{r_J} \\ &\quad + M_{st}\vec{r_{st}} + M_u\vec{r_u} + M_{nep}\vec{r_{nep}} + M_{ce}\vec{r_{ce}} + M_{pl}\vec{r_{pl}} + M_e\vec{r_e} \end{aligned}$$

If we take Jupiter along +ve axis, we will asssume all other planets and sun on the negative side

$$\begin{aligned} \therefore M_{\odot}r_{\odot} &= M_J r_J - M_{me}r_{me} - M_v r_v - M_{\oplus}r_{\oplus} - M_{ma}r_{ma} - M_{st}r_{st} \\ &\quad - M_u r_u - M_{nep}r_{nep} - M_{ce}r_{ce} - M_{pl}r_{pl} - M_e r_e \\ 1.989 \times 10^{30} r_{\odot} &= 1.898 \times 10^{27} \times 7.783 \times 10^{8} - 3.301 \times 10^{23} \times 5.791 \times 10^{7} \\ &\quad - 4.867 \times 10^{24} \times 1.082 \times 10^{8} - 5.972 \times 10^{24} \times 1.496 \times 10^{8} \\ &\quad - 6.417 \times 10^{23} \times 2.279 \times 10^{8} - 5.683 \times 10^{26} \times 1.427 \times 10^{9} \\ &\quad - 8.681 \times 10^{25} \times 2.871 \times 10^{9} - 1.024 \times 10^{26} \times 4.498 \times 10^{9} \\ &\quad - 9.470 \times 10^{20} \times 4.137 \times 10^{8} - 1.309 \times 10^{22} \times 5.906 \times 10^{10} \\ &\quad - 1.660 \times 10^{22} \times 1.018 \times 10^{11} \end{aligned}$$

$$\begin{aligned}1.989r_{\odot} &= 1898 \times 778.3 - 3.301 \times 5.791 - 4.867 \times 108.2 - 5.972 \times 149.6\\ &\quad - 6.417 \times 22.79 - 568.3 \times 1427 - 86.81 \times 2871\\ &\quad - 102.4 \times 4498 - 0.9470 \times 0.4137 - 1.309 \times 590.6\\ &\quad - 1.660 \times 1018\end{aligned}$$

ignoring all terms less than 1000 (as per least count of the data)

$$\begin{aligned}1.989r_{\odot} &\approx 1.477 \times 10^6 - 0.001 \times 10^6 - 0.811 \times 10^6 - 0.249 \times 10^6\\ &\quad - 0.461 \times 10^6 - 0.001 \times 10^6 - 0.002 \times 10^6\\ &\approx -0.048 \times 10^6\,\text{km}\\ \therefore r_{\odot} &\approx -25\,000\,\text{km}\end{aligned}$$

The negative sign indicates we slightly over-compensated by putting all other planets on the side of the Sun. Moving any of the gas giants on the side of the Jupiter will clearly tilt the balance the other way. So the optimal solution would be to move all the smaller bodies (Earth and everything smaller than that) on the Jupiter side.

Thus, finally we want Saturn, Uranus, Neptune and Sun on one side of the C.M. and Jupiter as well as all other listed smaller planets and dwarf planets on the other side of C.M.

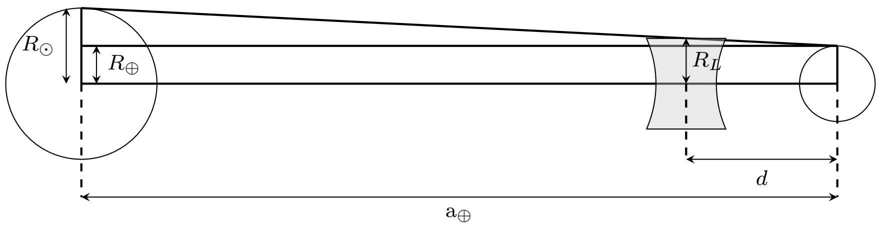
With that configuration, the centre of mass will be about 21 000 km from the centre of the Sun on the side opposite to Jupiter. i.e. Sun and Jupiter should be on same side of the centre of mass.

3. Ameya has come up with an ambitious plan to set up a giant lens at an appropriate place between the Earth and the Sun to reduce the flux of light falling on all points on the Earth.
   (a) (2 marks) What type of lens should be used for this purpose?
   (b) (2 marks) Where should we put the lens so that this reduction in intensity is permanent?
   (c) (3 marks) What should be the minimum diameter of this lens?
   (d) (6 marks) Assuming that we want to reduce the temperature of the Earth by 2 °C, what should be the focal length of this lens?
   (e) (5 marks) In reality, it is impractical to make a single lens of such a diameter. So Ameya's plan B is to replace this single lens by an array (like a net with lens at every node) of lenses of diameter 1 m and focal length 10 m. How many such lenses are required to be placed at the same distance, in a circular area of same diameter as the bigger lens to produce the same temperature change?
   (f) (2 marks) If we want to replace the lenses in part (e) by opaque spheres of diameter 1 m, what would be the corresponding drop in temperature?

Solution:

(a) We need some of the solar radiation incident on the Earth to diverge away. So a diverging, i.e. **Concave** lens should be used. Alternatively, a convex lens with shortfocus can also give same effect. [Such a solution with clear justification is also acceptable].

(b) As we want the lens to always remain between the Sun and Earth, it would make sense to keep it at 1st Lagrangian point (L1). Approximate distance of this point from the Earth is $d \approx 0.01\,\text{AU}$.[Alternate valid considerations are also acceptable.]



(c)

Using similar triangles

$$\frac{R_\odot - R_\oplus}{a_\oplus} = \frac{R_L - R_\oplus}{d}$$

$$\therefore R_L = \frac{d}{a_\oplus}(R_\odot - R_\oplus) + R_\oplus$$

$$\therefore R_L = 0.01(6.955 \times 10^8 - 6.371 \times 10^6) + 6.371 \times 10^6$$

$$\boxed{R_L = 1.326 \times 10^7\,\text{m} \approx 2.082 R_\oplus}$$

(d) As Earth is assumed to be in thermal equilibrium, with equillibrium temperature $T_{\text{eff}} = 5\,°\text{C} = 278\,\text{K}$. (acceptable range $0\,°\text{C}$ to $30\,°\text{C}$).

$$\pi R_\oplus^2 \frac{L_\odot}{4\pi a_\oplus^2} = 4\pi R_\oplus^2 \sigma T^4$$

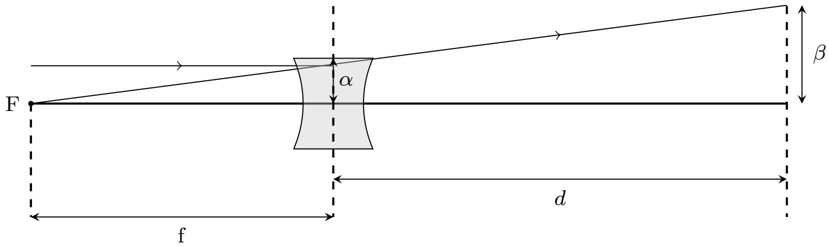
$$\frac{L_\odot}{4\pi a_\oplus^2} = \frac{4\pi R_\oplus^2 \cdot \sigma T^4}{\pi R_\oplus^2}$$

$$\therefore F_{recieved} = 4\sigma T^4$$

$$\therefore \frac{F_1}{F_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{276}{278}\right)^4$$

$$\frac{F_1}{F_2} = 0.972$$

Let α be the radius of the lens and β be the radius of area over which total flux recieved by lens is distributed. As $d << a_\oplus$, we can approximate all rays from the Sun to be parallel



$$\therefore \left(\frac{\alpha}{\beta}\right)^2 = \frac{F_1}{F_2} = 0.972$$

Using similar triangles,

$$\frac{\alpha}{f} = \frac{\beta}{f+d} \Rightarrow \frac{\alpha}{\beta} = \frac{f}{f+d}$$

$$\therefore \frac{f}{f+d} = \sqrt{0.972} = 0.986$$

$$\therefore \frac{1}{1+\frac{d}{f}} = 0.986, \Rightarrow \frac{d}{f} = \frac{1}{0.986} - 1$$

$$f = \frac{1.5 \times 10^9}{0.0145}$$

$$\boxed{\therefore f = 1.034 \times 10^{11}\,\mathrm{m}}$$

(e) Lens of diameter 1 m and focal length 10 m will diverge all the rays incident on it, and all those rays will miss the Earth so practically it behaves like an opaque object. If we use N number of such lenses to reduce temperature by 2 °C then ratio of non-opaque area to total area of a single huge lens must equal to ratio of fluxes.

$$\frac{(\pi\alpha^2 - N \cdot \pi\gamma^2)}{\pi\alpha^2} = \frac{F_1}{F_2}$$

where $\gamma = 1\,\mathrm{m}$ is radius of smaller lenses (given)

$$\therefore 1 - \left(\frac{N \cdot \pi\gamma^2}{\pi\alpha^2}\right) = \frac{F_1}{F_2}$$

$$\therefore \left(\frac{N \cdot 1^2}{(1.326 \times 10^7)^2}\right) = 1 - 0.972$$

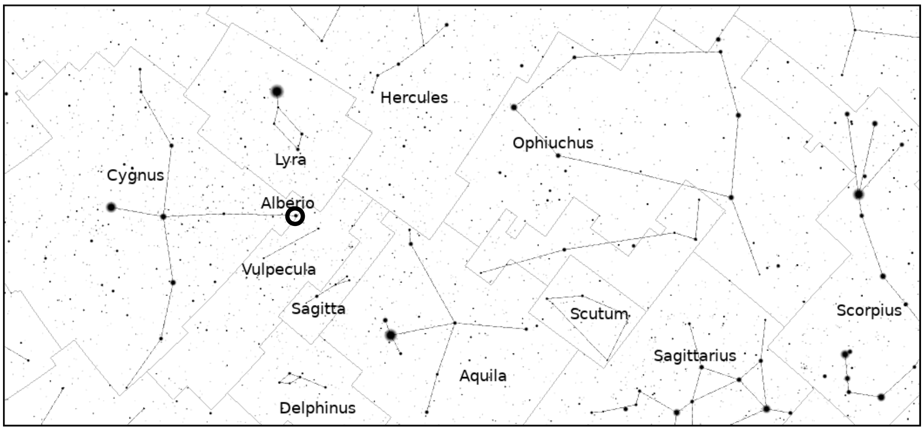
$$\therefore N = 0.028 \times (1.326 \times 10^7)^2$$

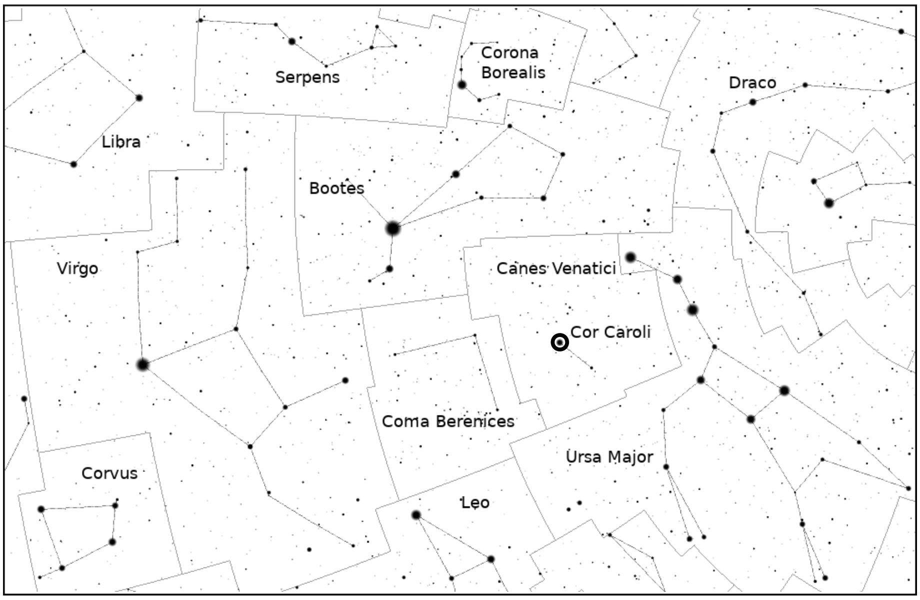
$$\boxed{\therefore N = 4.92 \times 10^{12}}$$

(f) In the previous part, as lenses behaves as opaque objects; we can replace them by any other opaque objects of same diameter and the corresponding temperature drop will be same as in case of lenses.
$\therefore$ If we replace lenses by opeque spheres of diameter 1 m then corresponding temperature drop will be of 2 °C.

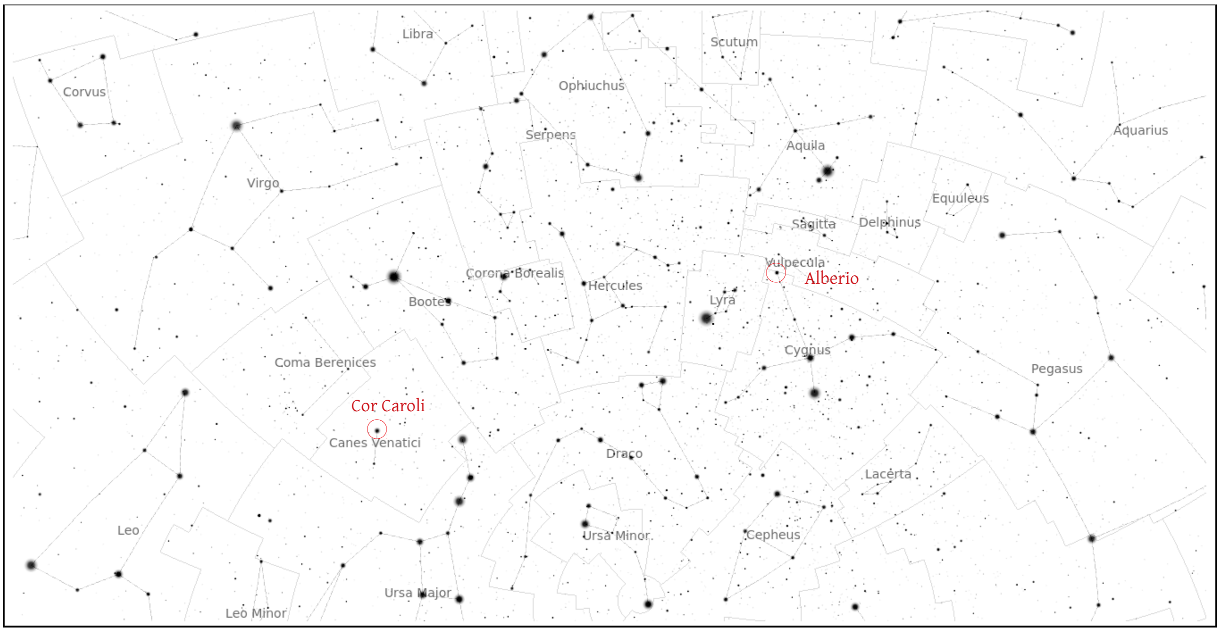
4. (14 marks) Vinita prepared the two maps given below and gave those to students for practice on a certain date. At that time, the sky looked like the skymap in your answersheet. Use the practice maps to identify following constellations / stars in the skymap provided in your answersheet.

Mark stars with circles around them and for constellations draw stick figures as shown in the practice maps below.
Sagitta, Cor Caroli, Alberio, Lyra, Corvus, Corona Borealis


Hercules
Ophiuchus
Lyra
Cygnus
Alberio
Vulpecula
Sagitta
Scutum
Scorpius
Sagittarius
Aquila
Delphinus


Serpens
Corona
Borealis
Draco
Libra
Bootes
Virgo
Canes Venatici
Cor Caroli
Coma Berenices
Ursa Major
Corvus
Leo

Solution:


Libra
Scutum
Corvus
Ophiuchus
Aquarius
Serpens
Aquila
Virgo
Equuleus
Sagitta
Delphinus
Vulpecula
Alberio
Corona Borealis
Hercules
Lyra
Bootes
Cygnus
Coma Berenices
Pegasus
Cor Caroli
Canes Venatici
Draco
Lacerta
Leo
Ursa Minor
Cepheus
Ursa Major
Leo Minor

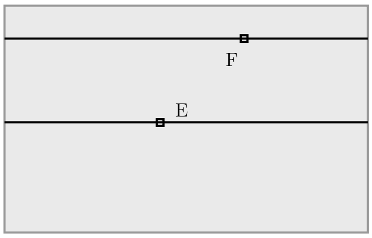
5. The maps of the Earth which you typically find in textbooks are printed with following criteria:

- All the latitudes and longitudes are straight lines.
- If any two points on the Earth along any longitude or along the equator have equal angular separation then on the map they will have equal linear distance.

Prathyush has a globe of Earth of radius R. He also has a map of Earth with a projection described above. He noticed that the length of equator in this map as well as on the globe was exactly the same.

(a) (3 marks) Find the ratio of the total surface area of the map A_{map} to the area of the globe A_{sph}.

(b) (5 marks) Two small squares E and F of identical size are drawn on this map (see the figure below). Square E lies very near to the equator, while square F lies at a latitude of 60°. Find the ratios $\frac{A_{E|map}}{A_{E|sph}}$, $\frac{A_{F|map}}{A_{F|sph}}$ and $\frac{A_{F|sph}}{A_{E|sph}}$
where, $A_{E|sph}$ corresponds to area of E as measured on the globe
$A_{E|map}$ corresponds to area of E as measured on the map.
$A_{F|sph}$ corresponds to area of F as measured on the globe
$A_{F|map}$ corresponds to area of F as measured on the map.



(c) (2 marks) A large square is drawn on the map such that its sides are u units ($u \sim R/4$). Lower side of this square is located on the equator. Draw approximate shape of the projection of this square as it appears on the globe.

Solution:

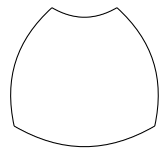
(a) The area of the Earth (sphere) is $A_{sph} = 4\pi R^2$
The area of the plane is $A_{map} = l \times b$
The length of the rectangle is the circumference of the sphere, $2\pi R$
The breadth of the rectangle is equal to half of the circumference $b = \pi R$
Hence, the ratio,
$$\frac{A_{map}}{A_{sph}} = \frac{\pi R \times 2\pi R}{4\pi R^2} = \frac{\pi}{2}$$

(b) In this transformation, there would be hardly any streching for a tiny square located at equator. Thus, $\frac{A_{E|map}}{A_{E|sph}} \approx 1$
For the square at the latitude of 60°, the vertical side remains unchanged (a), but the horizontal sides get stretched on the map. On the globe, the horizontal side will become, $l = a \cos 60°$.

$$\therefore \frac{A_{F|map}}{A_{F|sph}} = \frac{a^2}{a \times a\cos 60^\circ} = \frac{1}{\cos 60^\circ} = 2$$

As, $A_{E|map} = A_{F|map}$, we have $\dfrac{A_{F|sph}}{A_{E|sph}} = \dfrac{A_{F|sph}}{A_{F|map}} \times \dfrac{A_{E|map}}{A_{E|sph}} = \dfrac{1}{2} = 0.5$

(c) This is equivalent of taking two fixed latitudes, two fixed longitudes and projecting that shape on the globe. As you can visualise, this projection will look like a trapezium, since the side near the equator and sides along the longitudes will remain the same, but the side towards the poles will be compressed.



6. (10 marks) Consider a cylindrical metallic wire of cross-sectional area A, length l, and resistance R_0 at ambient temperature T_0, which is also the temperature of the wire at the start of experiment. The resistance of the wire varies as a function of temperature $R = R_0\ e^{\alpha(T-T_0)}$. Assume it acts like a perfect blackbody with an indefinitely high melting point. The wire undergoes cooling only via radiation and its dimensions are unaffected by any change in temperature. Sketch the approximate temperature *vs.* time graph if the ends of the wire are maintained at a constant potential difference (V_0).

Solution:

It is assumed that the wire has perfect thermal conductivity and one can ignore radiation loss from the end points of the wire.

Heat generated in the wire in time interval Δt will be

$$I^2 R\Delta t = \frac{V^2}{R}\Delta t$$

If T is final temperature of the wire, the heat emitted in the form of radiation in time interval Δt will be

$$2\sigma l\sqrt{\pi A}(T^4 - T_0^4)\Delta t$$

Heat used to raise the temperature of wire by ΔT such that new temperature now is $T - \Delta T$) will be $= mc\Delta T$, where c is specific heat of the wire.
Now, Heat generated - Heat emitted = Heat used to raise temperature. When constant potential difference (V_0) is maintained across the wire,

$$\left(\frac{V_0^2}{R_0\ e^{\alpha(T-T_0)}}\right)\Delta t - \left(2\sigma l\sqrt{\pi A}(T^4 - T_0^4)\right)\Delta t = mc\Delta T$$

$$\left(\frac{V_0^2}{R_0\ e^{\alpha(T-T_0)}}\right) - \left(2\sigma l\sqrt{\pi A}(T^4 - T_0^4)\right) = mc\frac{\Delta T}{\Delta t}$$

$$\left(\frac{V_0^2\ e^{\alpha T_0}}{R_0}\right)\ e^{-\alpha T} - \left(2\sigma l\sqrt{\pi A}\right)(T^4 - T_0^4) = mc\frac{\Delta T}{\Delta t}$$

In the graph below, we show first term (x) by purple curve and second term (y) by green curve.

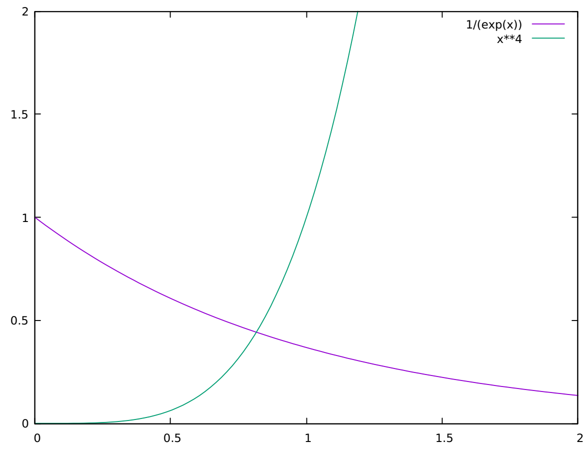


Figure 1: Graph of x and y $vs.$ T

at $t = 0$, $\quad T = T_0$, $\quad \therefore x = \dfrac{V_0^2}{R_0}$ $\quad$ and $\quad y = 0 \quad \therefore \dfrac{\Delta T}{\Delta t}$ is positive.

At $x = y$, $\quad T = T_C \quad$ and $\quad \dfrac{\Delta T}{\Delta t} = 0 \quad$.

Thus, the temperature will asymptotically reach to $T = T_C$

$\therefore$ The graph of T $vs.$ t will look like

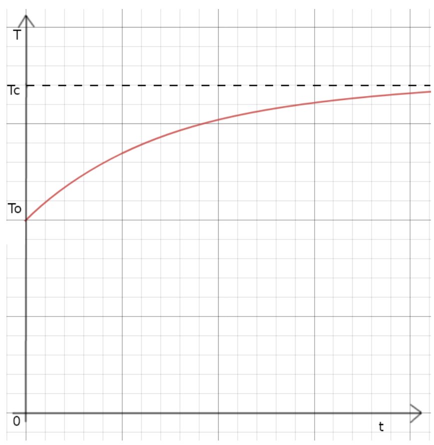


Figure 2: Graph of T $vs.$ t

[intially non-zero slope around $t = 0$, not a sigmoid shape, no concavity change, asymptote to T_C]

7. Separated from his friends, Yash landed up on an island on 23rd September. On the same day, with nothing better to do, he meticulously measured height of tide at every hour. This data is tabulated in Table 1 below.

   To model the tide height as a function of time, he made following simplifying assumptions;

Table 1: Data for height of tide at every hour (local time) on a 23rd September.

Time (hrs)	Height (m)		Time (hrs)	Height (m)		Time (hrs)	Height (m)
00:00	0.538		08:00	0.45		16:00	0.756
01:00	0.616		09:00	0.341		17:00	0.812
02:00	0.708		10:00	0.307		18:00	0.786
03:00	0.753		11:00	0.33		19:00	0.733
04:00	0.786		12:00	0.372		20:00	0.666
05:00	0.741		13:00	0.482		21:00	0.622
06:00	0.639		14:00	0.572		22:00	0.583
07:00	0.554		15:00	0.676		23:00	0.545

- Tides are caused **ONLY** by the gravitational pull of the Moon (i.e. ignore gravitational pull due to Sun).
- Ignore the effect of Earth's rotation on the tides.
- Ignore friction between the surface of the Earth and the water.

(a) (10 marks) Plot the given data.

(b) (4 marks) Using the plot, find the Sun-Earth-Moon angle.

(c) (2 marks) Find the date of the nearest New Moon.

(d) (6 marks) Later he explored interiors of the island, including a large cave system, and had erratic eating and sleeping schedule. As a result, when he returned to the beach, he had lost track of time. He again started measuring tide height to get a sense of date. However, this time, he was also busy in other activities such as food gathering and raft making. So his readings, as given in Table 2, are taken only sporadically. Find the height of the tide at 16:00 hours on that day, by overplotting this data on the same graph paper.

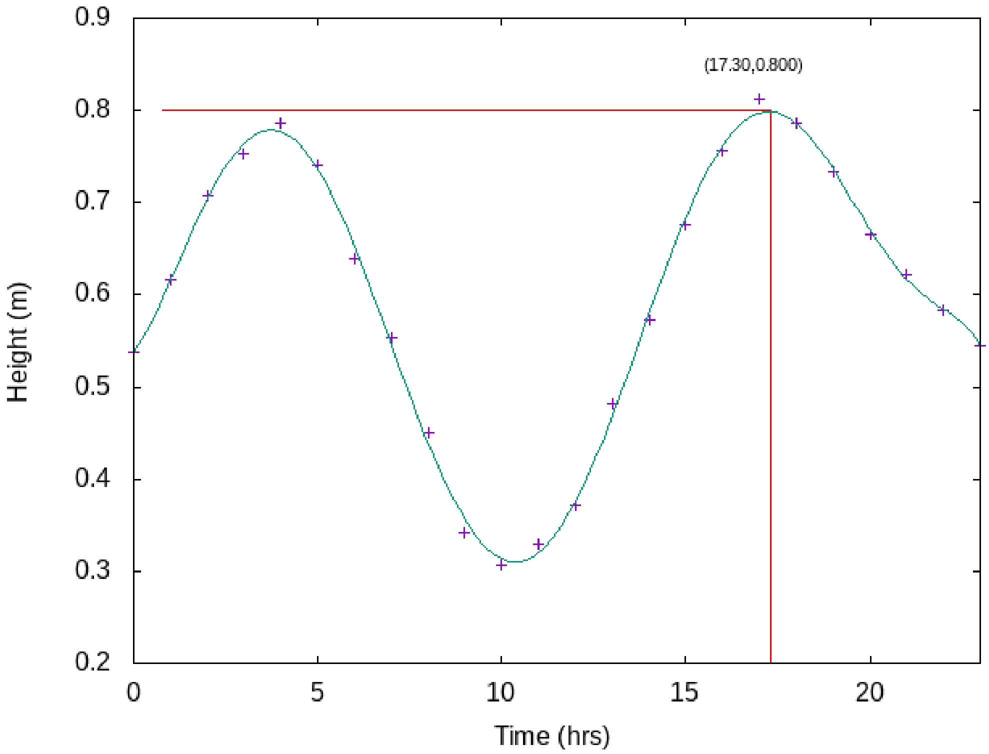
(e) (2 marks) Which is the date corresponding to Table 2?

Table 2: Height versus time data

Time (hrs)	Height (m)		Time (hrs)	Height (m)
00:00	1.014		12:00	0.748
01:00	1.079		13:00	0.855
02:00	1.033		14:00	0.844
03:00	0.905		18:00	0.279
07:00	0.255		19:00	0.213
08:00	0.235		20:00	0.203
09:00	0.322		21:00	0.307
10:00	0.445		23:00	0.682

Solution:

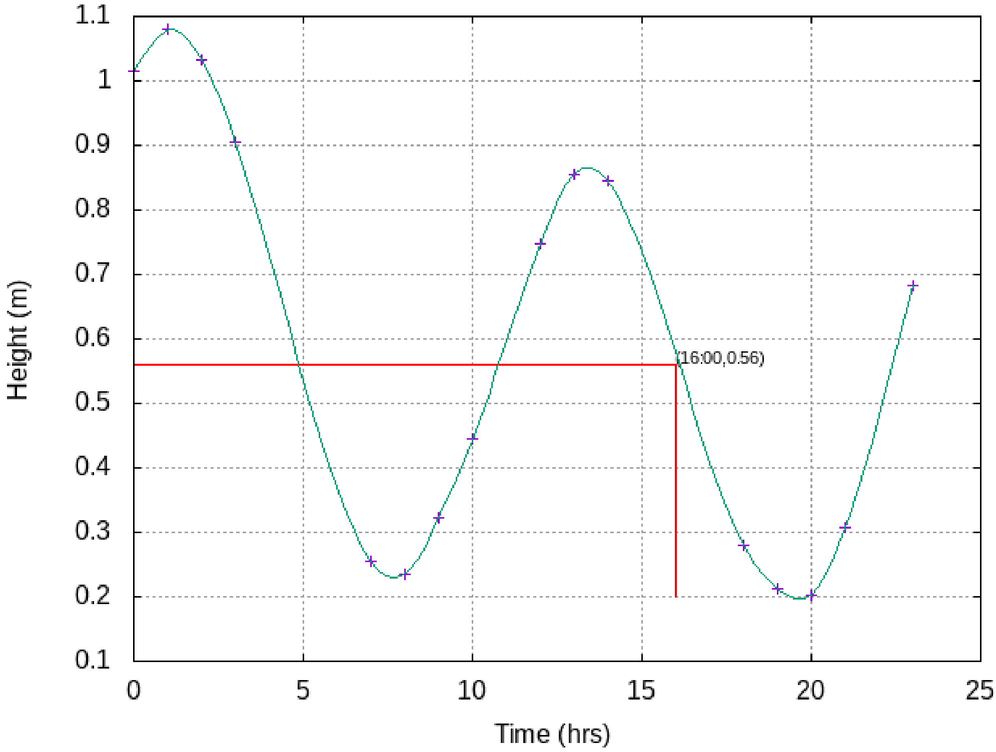
(a) Below is the graph. Marks will be given if all protocols of graph plotting axis label, scale, clear identification of points, smooth curve etc. are followed.



(b) In the graph, check the time when the height of tide is maximum. One can see that there is one maxima at 17:00 hours. This is when the Moon is exactly on meridian of the island. There is another maxima at around 04:00 hours which, is lesser than the one at 17:00 hours. This is when the Moon is exactly on the opposite side.
At 17:00 the Moon is at the meridian and the Sun was at meridian at local noon.
The Earth rotates at a rate of 15° per hour, and this being an equinox day, the Sun will also have the same rate.
Therefore the Sun will be 75° from the meridian. As nothing is specified about position of the lunar node, one may ignore that correction. Hence, the Sun-Earth-Moon angle is 75°.

(c) The New Moon occurs when the angle between the Sun-Earth-Moon is 0°.
Also, every day the Moon rises around 50 minutes late.
(Alternatively, Moon need 29.53 days to complete 360° around the Earth).
Thus, the Moon moves ahead by about 12.2° every day.
Present angle of 75° implies difference between the day of arrival and new Moon is about 6 days.
Next we notice that the Moon is setting after the Sun. This implies that it is waxing fortnight. Hence it is 6 days **AFTER** the new Moon.
Thus, the date of new Moon is **17$^{\text{th}}$ September**.

(d) See the plot below;



It is clear from the earlier plot and this plot that between any two turning points, the curve is more or less a straight line, hence at 16:00 hours the height of the tide is about 0.56 m.

(e) Since the height of the tide is maximum at 1:00 am in the night, one can say that this is the day after the full Moon.
Thus, the date is, **3$^{\text{rd}}$ October**.