

Indian National Astronomy Olympiad – 2009

Junior Category
Model Solutions

Roll Number:

INAO – 2009

Duration: **Three Hours**

Date: 31st January 2009

Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 3 pages (6 sides).
- In Section A, there are 10 multiple choice questions with 4 alternatives out of which only 1 is correct. You get 3 marks for each correct answer and **-1 mark for each wrong answer**.
- In Section B, there are 4 multiple choice questions with 4 alternatives each, out of which any number of alternatives may be correct. You get 5 marks for each correct answer. No marks are deducted for any wrong answers. **You will get credit for the question if and only if you mark all correct choices and no wrong choices**. There is no partial credit.
- For both these sections, you have to indicate the answers on the page 2 of the answersheet by putting a \times in the appropriate box against the relevant question number, like this:

Q.NO.	(a)	(b)	(c)	(d)		Q.NO.	(a)	(b)	(c)	(d)
22	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	OR	35	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Marking a cross (\times) means affirmative response (selecting the particular choice). Do not use ticks or any other signs to mark the correct answers.

- In Section C, there are 5 analytical questions totalling 50 marks.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- No calculators are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.

Useful Physical Constants

Mass of the Earth	$M_E \approx 5.97 \times 10^{24} \text{ kg}$
Radius of the Earth	$R_E \approx 6.4 \times 10^6 \text{ m}$
Mass of the Sun	$M_\odot \approx 1.99 \times 10^{30} \text{ kg}$
Radius of the Sun	$R_\odot \approx 7 \times 10^8 \text{ m}$
Radius of the Moon	$R_m \approx 1.7 \times 10^6 \text{ m}$
Speed of Light	$c \approx 3 \times 10^8 \text{ m/s}$
Astronomical Unit	1 A. U. $\approx 1.5 \times 10^{11} \text{ m}$
Gravitational Constant	$G \approx 6.67 \times 10^{-11} \text{ m}^3/(\text{Kg s}^2)$
Gravitational Acceleration on the Earth	$g \approx 9.8 \text{ m/s}^2$
Gravitational Acceleration on the Moon	$g_m \approx 1.6 \text{ m/s}^2$

Space for Rough Work

Section A: (10 questions × 3 marks each)

1. If $a^x = b^y = c^z$ and $b^2 = ac$, then $y = ?$

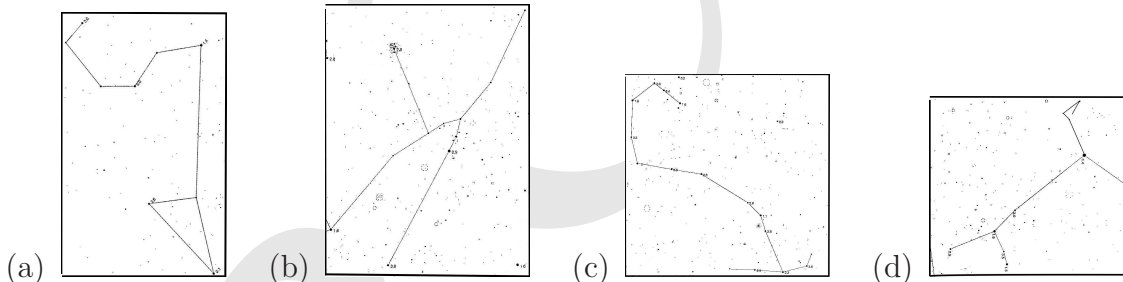
- (a) $\frac{2xz}{x+z}$ (b) $\frac{xz}{x+z}$ (c) $\sqrt{2xz}$ (d) \sqrt{xz}

Solution:

$$\begin{aligned} a &= b^{\frac{y}{x}} \\ c &= b^{\frac{y}{z}} \\ b^2 &= ac = b^{\frac{y}{x}} b^{\frac{y}{z}} \\ &= b^{\frac{y}{x} + \frac{y}{z}} \\ 2 &= \frac{y}{x} + \frac{y}{z} \\ &= \frac{y(x+z)}{xz} \\ y &= \frac{2xz}{z+x} \end{aligned}$$

Ans = (a)

2. Each of the figures below, depict a constellation. Find the odd one out.



Solution: The constellations are (a) Leo (b) Taurus (c) Scorpio (d) Canis Major. First three are zodiac signs whereas the fourth one is not.
Ans = (d)

3. Gravitational force between two identical uniform solid gold spheres of radius r each in contact is proportional to

- (a) r^4 (b) r^2 (c) $\frac{1}{r^2}$ (d) $\frac{1}{r^3}$

Solution: The distance between two spheres = $2r$ and masses are the same

$$\begin{aligned} \Rightarrow F &= \frac{GM^2}{(2r)^2} \\ &= \frac{G\left(\frac{4\pi r^3}{3} * \rho\right)^2}{4r^2} \\ \Rightarrow F &\propto \frac{r^6}{r^2} \\ F &\propto r^4 \end{aligned}$$

Thus ans = (a)

4. A copper cube and a wooden cube of same size are initially at room temperature. Then they are kept in an enclosure at 50°C . What can we conclude about the temperatures attained by both cubes after 5 hours?
- $T_{\text{copper}} > T_{\text{wood}}$ as thermal conductivity of copper is greater than that of wood.
 - $T_{\text{wood}} > T_{\text{copper}}$ as specific heat capacity of wood is greater than that of copper.
 - The temperatures will depend on the interplay between specific heat capacity and thermal conductivity of the materials.
 - Both temperatures will be practically the same, as they are in the enclosure for 5 hours.

Solution: Both copper and wooden cube will have same temperature, as that of the enclosure, because the time is sufficiently long to bring them in thermal equilibrium with their surroundings.

Ans = (d)

5. If the product of all the numbers from 1 to 100 is divisible by 2^n , then what is the maximum possible value for n ?
- (a) 128 (b) 97 (c) 64 (d) 87

Solution: There are 50 numbers between 1 to 100, which are divisible by at least first power of 2.

There are 25 numbers, which are divisible by at least second power of 2, i.e. 4. However, as they are already counted once in previous step, we count them again for only single power of 2 in this step.

Continuing in the same fashion, there are 12 numbers, which are divisible by at least third power of 2, i.e. 8.

There are 6 numbers, which are divisible by at least fourth power of 2, i.e. 16.
 There are 3 numbers, which are divisible by at least fifth power of 2, i.e. 32.
 There is only 1 number divisible by sixth power of 2, i.e. 64.

Summing,

$(50 + 25 + 12 + 6 + 3 + 1) = 97$ is the number of times factor 2 appears in the product.

So 2^{97} is the highest power of 2, which will be a factor of the product of all the numbers from 1 to 100.

Ans = (b)

6. A repairman on the T. V. tower finds his water bottle leaking at the rate of 5ml per second. He drops the bottle and it reaches the ground straight. If he was at a height of 125m at that time and there was 200ml of water left in the bottle, the amount of water left in the bottle (neglecting air resistance) just before it hit the ground was

- (a) 175ml (b) 50ml (c) 100ml (d) 200ml

Solution: Ans = (d).

The force acting on a bottle and water in it, is the gravitational force (constant acceleration g). Therefore because of this free fall motion, bottle and water containing in it, will have same velocity.

Hence just before hitting the ground, water bottle will contain the same amount, 200ml of water in it.

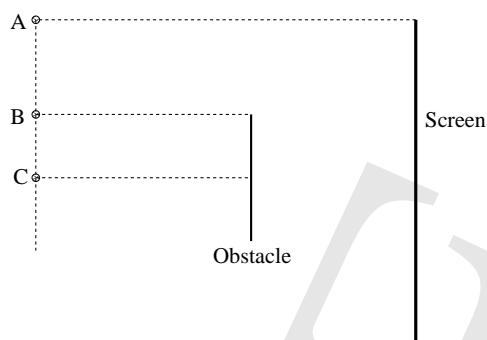
Note: In the actual question paper, there was a typographic error with height of tower specified as 125km rather than 125m. However, as the solution is independent of height, answer doesn't change.

7. In which of the following cities, your shadow will be the shortest, on the 15th of June?
 (a) Delhi (b) Bhopal (c) Bangalore (d) Thiruvananthapuram

Solution: The latitude of Bhopal is closest to the Tropic of Cancer. On the 15th June (summer solstice – 21st June), the Sun will be almost at zenith and therefore there we can see our shortest Shadow.

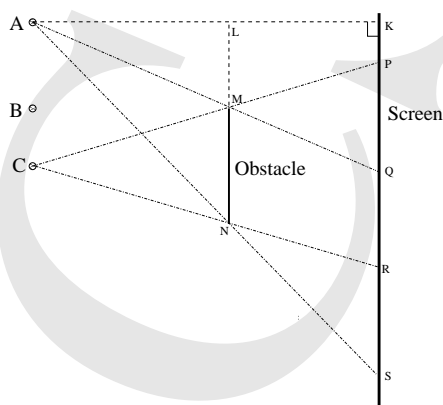
Ans = (b).

8. In the following figure, A, B, C are three light source positions with respect to the obstacle and the screen. Which of these light source positions will result in the longest shadow of the obstacle on the screen?



- (a) A
- (b) C
- (c) A and C form shadows of same length, while B forms a smaller shadow.
- (d) All the three light sources will form shadows of same length.

Solution: Ans = (d)



From the above figure, PR is the shadow because of light source C and QS is the shadow because of the light source A.

Now line AK is perpendicular to screen from A, which meets line MN at L .

Since $MN \parallel KS$,

$\therefore \triangle AMN$ and $\triangle AQS$ are congruent ,

which means $\frac{MN}{QS} = \frac{AL}{AK}$.

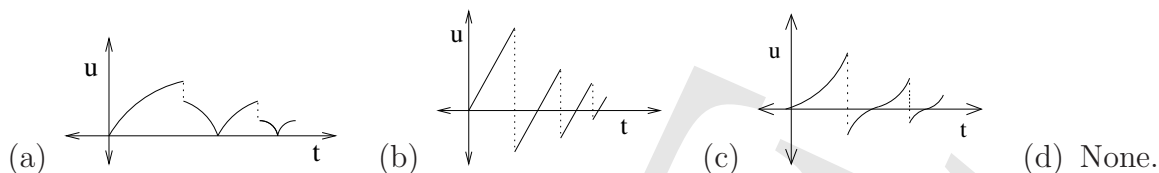
Similarly, $\triangle CMN$ & $\triangle CPR$ are congruent,

which means $\frac{MN}{PR} = \frac{AL}{AK}$.

$\implies QS=PR$.

Therefore, shadows of the obstacle formed by either light source A or C are the same. In the same manner, we can prove that the shadow formed by the light source B is also of the same length.

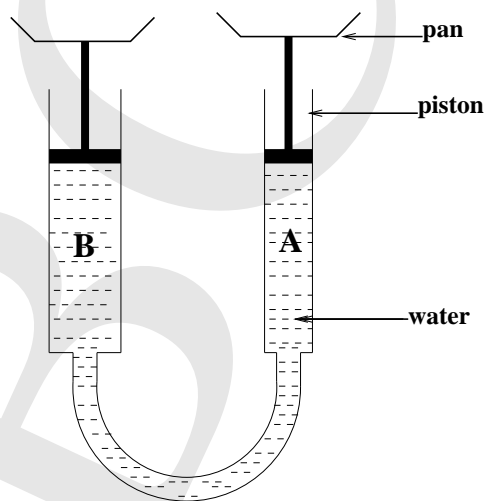
9. Which of the following represents the correct speed-time graph, for a ball bouncing frequently from a fixed surface?



Solution: Ans = (d).

From the kinematical equation of motion $v = (u + at)$, we can say that the speed-time graph should be a linear one. So options (a) and (c) are not correct. Also since speed can not assume negative values, even option (b) is incorrect. In fact, (b) represents correct velocity-time graph in case of inelastic collisions.

10. Two glass tubes filled with water are held vertical and connected by a plastic tube as shown in the figure. Pans are mounted on top of each piston such that $(\text{weight of the piston} + \text{pan})_A = (\text{weight of the piston} + \text{pan})_B$ radius of the piston A = 1.0cm and radius of piston B = 1.5cm. A 30.0gm of mass is added in pan B, what is the mass required in pan A to balance 30.0gm in pan B?



- (a) 67.5gm (b) 30.0gm (c) 13.3gm (d) 24.0gm

Solution: Ans = (c)

Let, mass in pan A = m_1 , mass in pan B = $m_2 = 30\text{gm}$.
radius of the piston A = $r_1 = 1.0\text{cm}$ and radius of piston B = $r_2 = 1.5\text{cm}$.

Now pressure in both tubes should be the same.

$$\begin{aligned} \therefore P &= \frac{F_1}{A_1} = \frac{F_2}{A_2} \\ \frac{m_1 g}{\pi r_1^2} &= \frac{m_2 g}{\pi r_2^2} \\ \frac{m_1}{1} &= \frac{30}{1.5^2} \\ \therefore m_1 &= \frac{30}{2.25} \\ \therefore m_1 &= \frac{30}{\frac{9}{4}} \\ \Rightarrow m_1 &= 13.3gm. \end{aligned}$$

Hence pan A required 13.3gm of mass to balance pan B.

Section B: (4 questions × 5 marks each)

11. Which of the following observations support the statement that “Every system tends to adjust by itself to have minimum Potential Energy”.
- Andromeda galaxy and Milky Way galaxy are approaching each other.
 - Two unlike, free charges move towards each other.
 - External work is required to compress a spring.
 - A powerful magnet can deflect a compass needle from equilibrium position.

Solution: Ans = (b), (c) and (d).

Andromeda Galaxy and the Milky Way are moving around the centre of our local group of galaxies. In the course of this motion, they just happen to be coming closer to each other. Their mutual gravitational attraction does not play any significant role in this motion.

12. In one of the truly revolutionary finds of the 20th century, Howard Carter discovered discovered tomb of Egyptian Pharaoh (emperor) Tutankhamen in 1922. Along with the mummy following items were also removed from the tomb. Which of these items could have been carbon dated to fix the period of the Pharaoh?
- Fragments of glass
 - Bronze Razor
 - Dried Fruits
 - Leather Shoe

Solution: Carbon dating relies on the fact that all living objects have ^{12}C and ^{14}C in a fixed ratio to each other. The radioactive ^{14}C keeps decaying to ^{12}C , however, it is replaced by food intake consumed by living organisms. After death, the decay process continues, but there is no replacement for decayed ^{14}C . Thus the ratio starts changing.

In the list above, the first two are not made from any organic / living substances. Thus, they cannot be used for carbon dating.

Ans = (c) and (d).

13. There is a regular bus service between Pune and Mumbai (180km apart) at every hour from both the ends, from 4am to 11pm. These busses run at average speed of 45km/hr. Taxies also run on the same route at 60km/hr with regular interval of 30min from 5am to 10pm. Following statements are based upon the number of taxies or busses crossed (not overtaken) only during travelling i.e. excluding instances of arrival and departure. Select the correct statement(s).

- (a) First bus crosses 6 taxis.
- (b) Last taxi crosses 5 buses.
- (c) Bus left at 8pm crosses 10 taxis.
- (d) Taxi left at 12 noon crosses 6 buses.

Solution: Ans = (a), (b), (c) and (d)

Bus covers the distance in 4 hours, taxi in 3 hours.

First ST bus is at 4am and it will reach its destination at 8am. Thus, during the journey, it will meet all taxies to have started from the other city from 5 am till 7:30 am. i.e. 5:00, 5:30, 6:00, 6:30, 7:00, 7:30 = 6 taxis

Last taxi starts at 10:00 pm, by which 6:00 pm bus would have arrived at the bus station. It reaches well past 11:00 pm (time of last bus). Thus, it will meet all the buses after the 6:00 pm bus. i.e. 7:00, 8:00, 9:00, 10:00, 11:00 = 5 busses.

The taxi leaving at 5:00 pm would have arrived by 8:00 pm. Thus bus will meet all taxies from 5:30 pm till the last taxi which leaves at 10:00pm i.e. 5:30, 6:00, 6:30, 7:00, 7:30, 8:00, 8:30, 9:00, 9:30, 10:00= 10 taxis

The 8:00 am bus would have arrived by noon. Thus the taxi meets 9:00am, 10:00am, 11:00am, 12:00noon, 1:00pm, 2:00pm buses on the way = 6 buses

14. Which of the following statement(s) is(are) useful, in estimating distances in the Universe?
- (a) Some time Venus can be seen transiting over the solar disc.

- (b) Stars with no proper motion appear to change their position in the sky when viewed six months apart.
- (c) Stars exhibit Doppler shift.
- (d) All supernovae of Type Ia, have same absolute brightness.

Solution: With Doppler shift we can estimate the velocity of stars but not the distance.

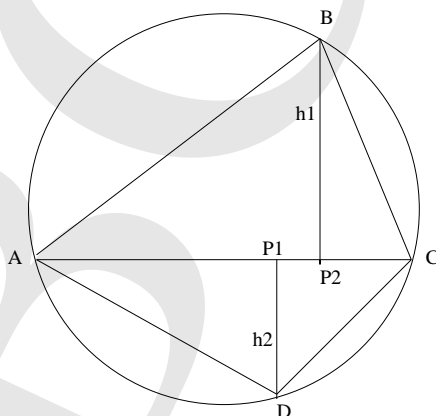
The Earth-Sun distance was successfully estimated for the first time using Venus transit method. Option (b) talks of parallax method. The absolute magnitudes of Supernovae is useful standard candle for cosmological distances.

Hence, ans= **(a)**, **(b)** and **(d)**.

Section C: Analytical Questions

- α. (8 marks) What will be area of the largest cyclic quadrilateral that can be inscribed in a given circle? Justify your answer qualitatively (formal proof not necessary).

Solution: Divide any quadrilateral ABCD, inscribed in a circle, into two triangles $\triangle ABC$ and $\triangle ADC$, as shown.



The area of the triangle is given by the formula,

$$\frac{1}{2} \times \text{base} \times \text{height}.$$

The common base of the two triangles $\triangle ABC$ and $\triangle ADC$ is given by \overline{AC} .

The total area, of the quadrilateral will be sum of the areas of the two triangles i.e.,

$$\frac{1}{2} \times l(\overline{AC}) \times (h_1 + h_2)$$

To maximize the total area, $l(\overline{AC})$ as well as $(h_1 + h_2)$ should be maximized. Now the maximum possible length that can be fitted inside a circle has to be its diameter, d .

Consequently, the total area of the quadrilateral ABCD would be maximum if, $(h_1 + h_2) = d$ and $l(\overline{AC}) = d$

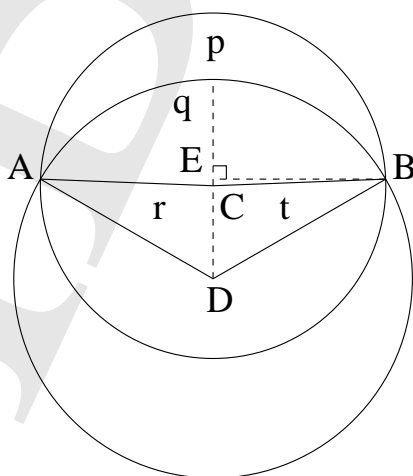
Thus, the maximum area can be $\frac{1}{2}d^2$.

Noting that the the base and height are perpendicular to each other, it is clear that the said quadrilateral is a square.

- If correct area with no justification: **2 marks.**
- If correct area with incorrect justification: **3 marks.**
- If justification only considers cyclic rectangles: **4 marks.**
- If correct justification, but minor error resulting in wrong area: **7 marks.**
- Any correct method will receive full consideration.

- β . (12 marks) Jayshree claimed that she saw a solar eclipse when the size of the solar disk was $26'$ and that of the lunar disk was $30'$. She also claimed that at the time of the maximum eclipse, distance between the centres of the two disks was $7'$. Qualitatively show that she could not have observed a total eclipse. Find the percentage of the solar disk covered at the time of the maximum eclipse. (Given: $\cos^{-1}(\frac{1}{26}) \approx 0.49\pi$ rad).

Solution: At the time of the maximum eclipse, the centres were $7'$ away from each other. However, the radius of the solar disk is smaller than that of the lunar disk by just $2'$. Thus, she must not have viewed the total solar eclipse. Let us find percentage of the maximum partial eclipse. **(1 mark)**



(1 mark)

In the figure above, points A and B make an angle of θ_1 and θ_2 with centre of the Solar and the Lunar disks (i.e. C and D) respectively.

r_s denotes the solar radius, r_m the lunar radius and h the separation between the two centres at the closest approach (all in arcminutes).

$$A(\widehat{ACB}) = \pi r_s^2 \frac{\theta_1}{2\pi}$$

$$p + q = \frac{r_s^2 \theta_1}{2}$$

$$A(\widehat{ADB}) = \pi r_m^2 \frac{\theta_2}{2\pi}$$

$$q + r + t = \frac{r_m^2 \theta_2}{2}$$

where $r = A(\triangle ACD)$ and $t = A(\triangle BCD)$

Also from figure, $r = t$.

$$\Rightarrow p = \frac{r_s^2 \theta_1}{2} - \frac{r_m^2 \theta_2}{2} + 2A(\triangle ACD). \quad (2 \text{ marks})$$

Now using Hero's formula for area of a triangle,

$$A(\triangle ACD) = \sqrt{s(s - r_m)(s - r_s)(s - h)}$$

where,

$$r_s = 13'$$

$$r_m = 15'$$

$$h = 7'$$

$$s = \frac{r_s + r_m + h}{2}$$

$$\therefore s = \frac{13 + 15 + 7}{2} = \left(\frac{35}{2}\right)'$$

$$\therefore A(\triangle ACD) = \sqrt{\frac{35}{2} \times \frac{(35 - 30)}{2} \times \frac{(35 - 26)}{2} \times \frac{(35 - 14)}{2}}$$

$$= \sqrt{\frac{35}{2} \times \frac{5}{2} \times \frac{9}{2} \times \frac{21}{2}}$$

$$= \sqrt{\left(\frac{5 \times 7 \times 3}{4}\right)^2 \times 3}$$

$$= \frac{105}{4} \times \sqrt{3}$$

$$\approx \frac{(173.2 + 8.7)}{4}$$

$$\approx \frac{91}{2} \quad (2 \text{ marks})$$

Now to find θ_1 and θ_2 draw a perpendicular line BE to line CD.

Let $EC = x$ and $EB = y$.

\therefore In $\triangle ECB$,

$$x^2 + y^2 = r_s^2.$$

and in $\triangle EDB$,

$$(x + h)^2 + y^2 = r_m^2$$

$$2hx + h^2 + r_s^2 = r_m^2$$

$$\therefore x = \frac{r_m^2 - r_s^2 - h^2}{2h} = 0.5'$$

$$\therefore \cos\theta_1 = \frac{x}{r_s} = \frac{0.5}{13} = \frac{1}{26}$$

$$\Rightarrow \theta_1 = 0.49\pi \quad (2 \text{ marks})$$

Similarly,

$$\cos\theta_2 = \frac{x + h}{r_m} = \frac{0.5 + 7}{15} = \frac{1}{2}$$

$$\Rightarrow \theta_2 = \frac{\pi}{3} \quad (1 \text{ mark})$$

These are half angles only. Total angles are double these values.

$$\Rightarrow p = \frac{r_s^2 \times 2 \times \theta_1}{2} - \frac{r_m^2 \times 2 \times \theta_2}{2} + 2A(\triangle ACD)$$

$$\approx 169 \times 0.49\pi - 225 \times \frac{\pi}{3} + 2 \times \frac{91}{2}$$

$$\approx \pi \left(84 - 75 + \frac{91 \times 7}{22} \right)$$

$$\approx \pi \left(9 + \frac{637}{22} \right)$$

$$\approx \pi(9 + 28.95)$$

$$\approx 38\pi \text{ arcmin}^2 \quad (1.5 \text{ marks})$$

$$\text{and } A(\text{Sun}) = \pi r_s^2$$

$$= 169\pi \text{ arcmin}^2 \quad (0.5 \text{ marks})$$

\therefore The amount of solar surface covered by moon at the time of maximum eclipse

$$= \left(1 - \frac{p}{A(\text{Sun})} \right) \times 100\%$$

$$\approx \left(1 - \frac{38}{169} \right) \times 100\%$$

$$\approx \left(1 - \frac{2.923}{13} \right) \times 100\%$$

$$\approx (1 - 0.22) \times 100\%.$$

$$\approx 78 \%. \quad (1 \text{ mark})$$

γ . (8 marks) The famous Indian astronomer, Aryabhata, expressed the value of π in what we now know as continuing fractions i.e. $\pi = 3.1416 = a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$ where a , b , c , d are positive integers. Find a , b , c , d .

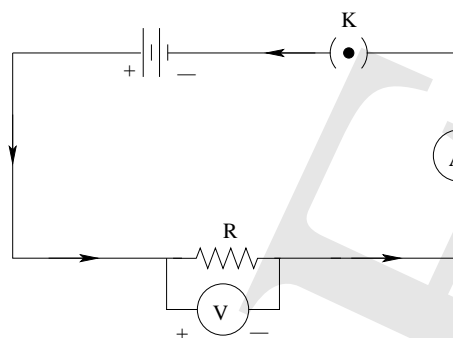
Solution:

$$\begin{aligned}
 3.1416 &= 3 + \frac{1416}{10000} \\
 &= 3 + \frac{1}{\frac{10000}{1416}} \\
 &= 3 + \frac{1}{9912 + 88} \\
 &= 3 + \frac{1}{7 + \frac{1}{\frac{88}{1416}}} \\
 &= 3 + \frac{1}{7 + \frac{1}{1408 + 8}} \\
 &= 3 + \frac{1}{7 + \frac{1}{16 + \frac{1}{\frac{88}{8}}}} \\
 &= 3 + \frac{1}{7 + \frac{1}{16 + \frac{1}{11}}}
 \end{aligned}$$

Therefore $a = 3$, $b = 7$, $c = 16$, $d = 11$.

- If attempted to solve polynomial: **1 mark**.
- If approximate reciprocals found correctly: **6 marks**.
- If approximate reciprocals found correctly and final answer is tallied back: **full marks**.
- For each wrong value out of a , b , c , d : **-1 mark each**.

δ. Mehul performed an experiment to verify Ohm’s law. He connected following circuit to measure voltage and current.



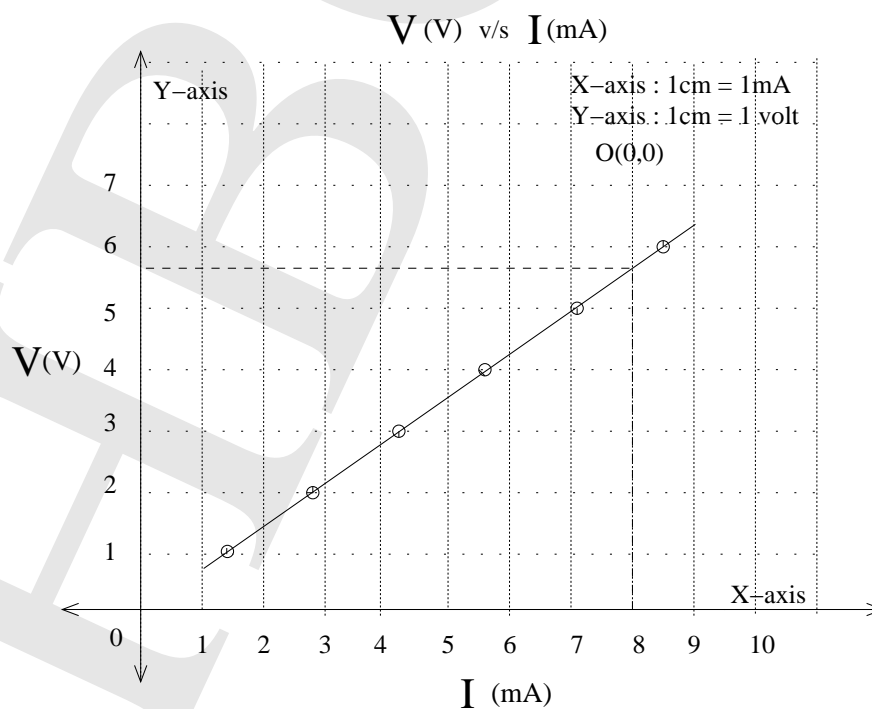
Here, R is the unknown resistance, V the voltmeter, A the ammeter and K is the key. He obtained following readings :

V (v)	1	2	3	4	5	6
I (mA)	1.40	2.83	4.26	5.68	7.11	8.54

- (9 marks) Plot appropriate graph to represent the data.
- (2 marks) Find the value of R .
- (1 mark) From the graph, what will be the voltage across the resistance when $I = 8\text{mA}$?

Solution:

(a) From the given observation table we have plotted voltage (v) versus current (mA) graph.



- Optimum amount of graph paper should be utilized. (1 mark)
- Choice of scale should be convenient. (1 mark)
- Optimum scale for voltage on larger axis: y-axis – 2 big sq. = 1V, x-axis – 1 big sq. = 1mA
- Optimum scale for current on larger axis: y-axis – 2 big sq. = 1V, x-axis – 2 big sq. = 1mA
- Points should be clearly marked. (1.5 mark)
- Chart Title, Axis Titles etc. should be properly written. (1 mark)
- The scales and preferably origin should be clearly specified at the top right corner. (1 mark)
- Points used for finding slope should be well separated, well marked and preferably not from the existing dataset. (1 mark)
- Line should be continuous and well balanced. (1.5 mark)
- 8mA point should be marked on the graph paper (as shown). (1 mark)

(b) Hence from the above graph, (2 marks)

$$\begin{aligned}
 \text{slope} &= \frac{(y_2 - y_1)}{(x_2 - x_1)} \\
 \Rightarrow R &\approx \frac{(4.2 - 0.7)V}{(6 - 1)mA} \\
 &\approx \frac{(3.5)}{(5 \times 10^{-3})} \Omega \\
 \therefore R &\approx 700 \Omega \quad (2 \text{ mark})
 \end{aligned}$$

(c) From the graph, for I = 8mA; V = 5.6V (1 mark)

- e. If the entire surface of the earth is covered using A4 size (size of your answer sheet) sheets of paper, what will be the total weight of paper used?

Solution: The total surface area of the Earth (2 marks)

$$\begin{aligned}
 A &= 4\pi R^2 \\
 &= 4 \times \pi \times (6.4 \times 10^8)^2 \text{ cm}^2
 \end{aligned}$$

Now normal size of A4 paper

$$a = (20 \times 30) \text{ cm}^2$$

To cover the whole earth we will require (3 marks)

$$\frac{4 \times \pi \times 6.4^2 \times 10^{16}}{20 \times 30} \text{ papers} \approx 82 \times 10^{14} \text{ papers}$$

Now thickness of a typical 100 page notebook is 1cm (or thickness of question paper + answer sheet was about 2mm). So thickness of a single paper = 0.01cm = 0.1mm. (1 mark)

∴ mass of a single A4 paper :

$$m = \text{volume} \times \text{density of a paper}$$

We take, density of a paper \approx density of wood = 0.5 gm/cc (1 mark)
(It should be definitely less than water as even crumpled paper floats on the water)

$$\begin{aligned} \therefore \text{mass of a paper} &\approx 600\text{cm}^2 \times 0.01\text{cm} \times 0.5\text{gm/cm}^3 \\ &\approx 3 \text{ gm} \quad (1 \text{ to } 5 \text{ gm acceptable}) \quad (1 \text{ mark}) \\ w &\approx 8.1 \times 10^{15} \times 3 \\ &\approx 2.4 \times 10^{16} \text{ gmwt} \\ &\approx 2.4 \times 10^{13} \text{ kgwt} \\ &\approx 24 \times 10^9 \text{ Tonne wt} \\ &\approx 2.4 \times 10^{14} \text{ N} \end{aligned}$$

∴ Total weight of paper used is about 24 Trillion Kgw. (2 marks)