Regional Mathematical Olympiad - 2025

Time: 3 hours **November 16, 2025**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **All questions carry equal marks. Maximum marks: 102.**
- **No marks will be awarded for stating an answer without justification.**
- **Answer all the questions.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Solve the following system of equations in nonnegative integers $a_1, a_2, \ldots, a_8$ where $a_i \neq 1$ for $i = 1, \ldots, 8$:

$$a_1a_2 = a_3 + a_4,$$
$$a_3a_4 = a_5 + a_6,$$
$$a_5a_6 = a_7 + a_8,$$
$$a_7a_8 = a_1 + a_2.$$

2. Let a, b, c be positive real numbers satisfying $abc = 1$. Prove that

$$\frac{2a^2}{a^3+1} + \frac{2b^2}{b^3+1} + \frac{2c^2}{c^3+1} \leq a^2 + b^2 + c^2.$$

3. Let $ABCDE$ be a convex pentagon in which $AB = AE$, $CB = CD$, and $\angle AED = \angle CDE = 90°$. Let the internal bisectors of $\angle EAB$ and $\angle DCB$ intersect at I, and let M be the midpoint of AC. Prove that $\angle MIC = \angle AIB$.

4. A frog is initially at $(0, 0)$ and it reaches $(n, 2)$, $n \geq 1$, using the following moves in any order several times:

   (i) $R = (1, 0)$, that is, if the frog is at (a, b) it goes to $(a + 1, b)$;
   (ii) $U = (0, 1)$, that is, if the frog is at (a, b) it goes to $(a, b + 1)$;
   (iii) $D = (1, 1)$, that is, if the frog is at (a, b) it goes to $(a + 1, b + 1)$.

   In how many ways can the frog go from $(0, 0)$ to $(n, 2)$, $n \geq 1$, using the above steps subject to the condition that steps of the type UU, DD are forbidden?
   (For example, for $n = 3$, $RDUR$, DRD are admissible paths, while DDR, $RUURR$ are not.)

5. Let ABC be an acute-angled triangle with $\angle BAC = 60°$ and $AB < BC < AC$. Let M, N be the midpoints of AB, AC respectively. Suppose BE, CF are altitudes, with E on CA and F on AB. Let X be the image of M under reflection in the midpoint of BF, and Y be the image of N under reflection in the midpoint of CE. Prove that XY bisects BC.

6. Define the sequence $\langle a_0, a_1, a_2, \ldots \rangle$ as follows: $a_0 = 49$ and $a_n = 10^{2^n} a_{n-1} - 1$ for $n \geq 1$. Show that $s(a_n^2) = n^2 + n + 7$ for all $n \geq 0$, where $s(m)$ denotes the sum of digits in base 10 representation of a nonnegative integer m.

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