Common mistakes made by contestants in INMO 2020

N.B: The contestants have been penalised for committing the following mistakes.

- **Question 1**
  - The points P, Q, A, and B are collinear.
  - The lines AX and AY are diameters of Γ_1 and Γ_2.
  - $XY//O_1O_2$; $CD//O_1O_2$; $XY//CD//O_1O_2$.
  - The points X, A, C are collinear.
  - The points D, A, Y are collinear.
- **Question 2**
  - A majority of the contestants ended up proving the converse i.e the claimed polynomial satisfies the hypothesis of the problem.
  - Many contestants assumed $x = \cos\theta + \sin\theta$ but failed to distinguish between $\sqrt{2 - x^2}$ and $-\sqrt{2 - x^2}$ as the value of $\cos\theta - \sin\theta$.
- **Question 3**
  - Answer cannot be outside the set $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Many claimed that the number of elements in the desired set is a number that does not belong to A.
  - Picking numbers at each step so as to keep the set minimal and capable for expressing $\{0, \ldots, i\}$, $i \in \{1, \ldots, 9\}$ may give **one** optimal set, but that doesn't prove optimality. Greedy is not necessarily optimal!
  - Number of sums we can form from a k-element set **is not** $\binom{k}{2}$. It is $\binom{k}{2} + k$. Do not forget sums of the form $a + a$, $a \in S$.
  - Since $1 = 0 + 1$ is the only way to split 1, your example set needs to have **both** 0 and 1.
  - Some students claimed that for $|S| = 4$, if $S = \{a_1, a_2, a_3, a_4\}$ then each of the sum $a_i + a_j$ correspond to a unique digit between 0 and 9, and so summing them up gives

    $$4(a_1 + a_2 + a_3 + a_4) = 45.$$

    Wrong! It would sum to $5(a_1 + a_2 + a_3 + a_4)$.
- **Question 4**
  - Use of induction in an inappropriate way. Assuming that $a_1 + a_2 + \ldots + a_n = 2n$ for every $n \geq 3$. It is given that this holds for some value of n, not for every n. If $a_1 + a_2 + \ldots + a_n = 2n$ for every n, then $a_1 = a_2 = \cdots = a_n = 2$.
  - Writing the inequality as

    $$\frac{1}{a_2 \ldots a_n} + \frac{1}{a_3 \ldots a_n} + \cdots + \frac{1}{a_n} + \frac{2}{a_1 a_2 \ldots a_n} \leq 1.$$

    Since $a_1 \leq a_2 \leq \cdots \leq a_n$, it suffices to prove that

    $$\frac{1}{a_1} + \frac{1}{a_1^2} + \cdots + \frac{1}{a_1^n} \leq 1 - \frac{1}{a_1^n}$$

    which holds if and only if $a_1 \geq 2$. But no way of getting $a_1 \geq 2$ from the conditions on $a_1, \ldots, a_n$.

- **Question 5**
  - Assuming the vertices of the frameable regular polygon ($n \geq 5$) will lie on adjacent parallel lines.
  - Drawing a figure and not giving any angle measurements or side lengths.
- **Question 6**
  - Getting an answer which is not a nonnegative integer.
  - Only considering one possible arrangement.
  - Covering the boards with 2 'layers' of strominoes. Not all arrangements are composed of two disjoint layers.
  - Not considering all possible coverings of the board with strominoes.

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