

Indian National Astronomy Olympiad – 2008

Senior Category
Model Solutions

Roll Number:

INAO – 2008
Duration: **Two and half Hours**

Date: 2nd February 2008
Maximum Marks: 100

Please Note:

- The question paper consists of two parts. In part A, there are 20 multiple choice questions with 3 marks for each correct answer and **-1 mark for each wrong answer**. In each question, only one of the four alternatives is correct. Mark the correct answer on the answer sheet provided separately. Mark a cross (X) in the corresponding box on the answer sheet.
- In part B, there are 4 analytical questions of 10 marks each. The answer to each question must be written in the blank space provided below each question.
- For the rough work, use the page(s) marked as rough sheet.
- Only non-programmable calculators are allowed.
- Return the ENTIRE question paper booklet and the answersheet back to the invigilator. **DO NOT TAKE THIS BOOKLET BACK WITH YOU.**

Please fill in all the data below correctly. The contact details provided here would be used for all further correspondence.

Full Name (BLOCK letters) Ms. / Mr.: _____

Male / Female Date of Birth (dd/mm/yyyy): _____

Name of the school / junior college: _____

Class: IX / X / XI / XII Board: ICSE / CBSE / State Board / Other

Full Residential address (include city and PIN code): _____

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HOMI BHABHA CENTRE FOR SCIENCE EDUCATION

Tata Institute of Fundamental Research

V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Instructions for using the answersheet:

- Write the name at the top of the answer sheet.
- On the left side there is space provided for roll number. Write your INAO roll number in the squares with exactly one digit per square.
- Below each of the digits of roll number, mark corresponding digit by a cross mark ('X'). i.e. if your roll number is 40001, then you will put X on 4, 0, 0, 0 and 1 in the corresponding columns.
- Use only black or blue pen to put 'X' marks on the answersheet. **Do not use any other ink or pencil.**

Useful Physical Constants

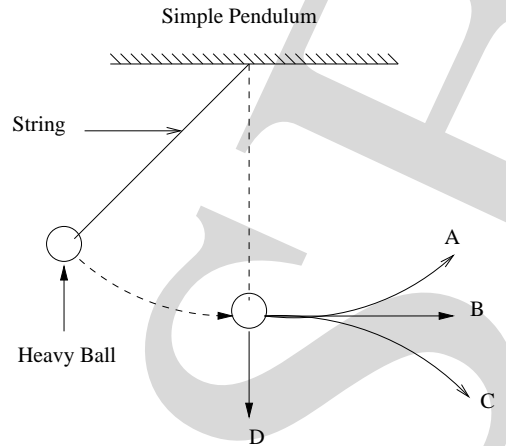
Mass of Earth	$M_E \approx 5.97 \times 10^{24} \text{ kg}$
Radius of Earth	$R_E \approx 6.4 \times 10^6 \text{ m}$
Mass of Sun	$M_\odot \approx 1.99 \times 10^{30} \text{ kg}$
Radius of Sun	$R_\odot \approx 7 \times 10^8 \text{ m}$
Speed of Light	$c \approx 3 \times 10^8 \text{ m/s}$
Astronomical Unit	1 A. U. $\approx 1.5 \times 10^{11} \text{ m}$
Gravitational Constant	$G \approx 6.67 \times 10^{-11} \text{ m}^3/(\text{Kg s}^2)$
Gravitational Acceleration	$g \approx 9.8 \text{ m/s}^2$
Speed of Sound (at room temperature in air)	$c_s \approx 340 \text{ m/s}$
Specific Heat of water	$C_w \approx 4.186 \times 10^3 \text{ J/kg}^\circ\text{C}$

Space for Rough Work

Part A: Multiple Choice Questions

- The dimensions of Boltzmann constant (k) are
 (a) $M^1 L^2 T^1$ (b) $M^1 L^2 T^{-1}$ (c) $M^2 L^1 T^{-1}$ (d) **$M^1 L^2 T^{-2}$**
 Note: Objections are raised over this question, because temperature dimensions are not specified. Thus, Question was removed from evaluation.
- P. One can see absorption lines in the Solar spectrum.
 Q. The core of the Sun has temperature of more than 1 million degree Celsius and the Solar surface has temperature of about 6000 degree Celsius.
 Which of the following options is correct?
 (a) Statement 'P' is correct but 'Q' is incorrect.
 (b) Statement 'P' is incorrect but 'Q' is correct.
 (c) **Both the statements are correct and 'Q' is the correct reason of 'P'.**
 (d) Both the statements are correct and 'Q' is not the reason of 'P'.
- When you stand on the ground, what is the distance of the horizon from you?
 (a) 500 km (b) **5 km** (c) 15 km (d) 50 km
- A regular barometer is thrown from the top of a building. If the barometer is freely falling, what will be the height of the mercury column?
 (a) **100 cm** (b) 76 cm (c) 50 cm (d) 0 cm
- P. Eclipses are not distributed evenly throughout the year, but happens only in certain months of a given year.
 Q. Orbit of the Moon (around the Earth) makes an angle of roughly 5 degrees to the orbit of Earth (around Sun).
 Which of the following options is correct?
 (a) Statement 'P' is correct but 'Q' is incorrect.
 (b) Statement 'P' is incorrect but 'Q' is correct.
 (c) **Both the statements are correct and 'Q' is the correct reason of 'P'.**
 (d) Both the statements are correct and 'Q' is not the reason of 'P'.
- For the Earth, if the perihelion were 147 million km, approximately what will be the aphelion for the Earth?
Aphelion: Point farthest from the Sun in the orbit of a body about the Sun.
Perihelion: Point nearest from the Sun in the orbit of a body about the Sun.
 (a) About 2 times the Perihelion, 300 million km

- (b) About 3 times the Perihelion, 450 million km
 (c) **Slightly more than the perihelion, about 155 million km**
 (d) Exactly the same as the perihelion, 147 million km
7. When the ball at the end of the string swings to its lowest point, the string is cut by a sharp knife as shown. Assuming no air resistance, what will be the path of the ball?

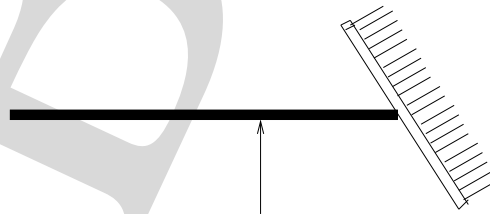


- (a) A (b) B (c) C (d) D

The answer is (c).

8. If we ever make contact with aliens, which of our fundamental units is likely to match theirs? (In other words, which of these units is universally fundamental?)
 (a) Kelvin (b) Light year (c) a.m.u. (Atomic Mass Unit) (d) **None of these**
9. P. In a dynamo, it is necessary that coil should be moving and magnet should be stationary and not vice versa.
 Q. Magnetic force is perpendicular to the direction of motion of charges.
 Which of the following options is correct?
 (a) Statement 'P' is correct but 'Q' is incorrect.
 (b) **Statement 'P' is incorrect but 'Q' is correct.**
 (c) Both the statements are correct and 'Q' is the correct reason of 'P'.
 (d) Both the statements are correct and 'Q' is not the reason of 'P'.
10. If a person beats drum on the Earth and an astronaut beats an identical drum in *space*, what will be the differences in the effects?
 (a) There will be no vibration in the drum in space.
 (b) **There will be vibration in space but no sound.**
 (c) The drum on Earth will vibrate for a longer time then the one in space.
 (d) There will be no difference in terms of the vibrations or sound.
11. An Olympiad student fails to get a medal. The angry team leader launches him into a rocket far into space & then throws him out. The student just grazes the upper edge of the atmosphere during his trajectory. He will...

- (a) **Go around the Earth**
 (b) Crash on the Moon
 (c) Get lost into deep space
 (d) Reach the Olympiad venue again
12. P: Gravitational force exerted by Saturn on a human being is approximately same as that exerted by another human being standing a few cm away.
 Q. Saturn has very low density.
 (Additional data: Mass of Saturn = 5×10^{26} kg, Distance of Saturn = 1.4×10^9 km)
- (a) Statement 'P' is correct but 'Q' is incorrect.
 (b) Statement 'P' is incorrect but 'Q' is correct.
 (c) Both the statements are correct and 'Q' is the correct reason of 'P'.
 (d) **Both the statements are correct and 'Q' is not the reason of 'P'.**
13. P. Temperature is not constant in an adiabatic process.
 Q. Adiabatic processes do not obey ideal gas equation.
 Which of the following options is correct?
- (a) **Statement 'P' is correct but 'Q' is incorrect.**
 (b) Statement 'P' is incorrect but 'Q' is correct.
 (c) Both the statements are correct and 'Q' is the correct reason of 'P'.
 (d) Both the statements are correct and 'Q' is not the reason of 'P'.
14. A star is seen rising from Calcutta ($23.5^\circ\text{N } 92^\circ\text{E}$) at 7:00 pm IST, at what time IST will it be seen to rise from Mumbai ($19^\circ\text{N } 72^\circ\text{E}$)?
- (a) 5:40 pm (b) 7:00 pm (c) 7:20 pm (d) **8:20 pm**
15. A broom with a long handle balances at its centre of gravity as shown in the figure. If you cut the broom into two parts through the centre of gravity and then weigh each part, which part will weigh more?

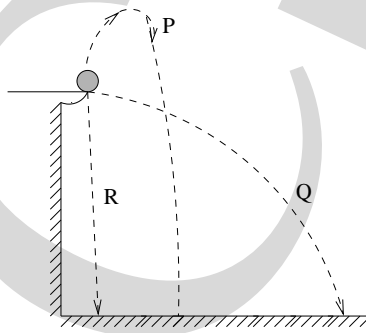


- (a) **The part with the broom will weigh more.**
 (b) The part without the broom will weigh more.
 (c) Both the parts will weigh the same.
 (d) It would depend on the weight of the broom.
16. Star A has temperature 4000°K and star B has temperature $40,000^\circ\text{K}$. If the two stars have roughly same radii, which of the following statements is **not** true?
- (a) B is more luminous than A.

- (b) A emits more in IR than in UV.
- (c) B emits more in UV than in IR.
- (d) **A emits more in IR than B.**

Note: In the actual question the word “NOT” was missing and the fact that both stars are assumed to have same radii was not explicitly stated. Thus, the question was removed from evaluation.

17. Every object exerts gravitational force on every other object - The *force* exerted by an object is higher if its *mass* is higher. Consider 2 magnets - a bigger magnet P and a smaller one Q. Which of the following will be true?
- (a) Magnet P will exert a greater magnetic force than Q.
 - (b) **The magnetic forces exerted by P and Q will be the same.**
 - (c) Magnet Q will exert a greater magnetic force than P.
 - (d) We cannot tell from the sizes, as gravity and magnetism are unrelated.
18. Three balls are thrown from the top of cliff along paths P, Q and R. If their initial speeds are the same and there is no air resistance, under what conditions will the balls strike the ground below with the same speed?



- (a) This will happen if the mass of each ball is the same.
 - (b) This will happen if the distance traveled by each ball is the same.
 - (c) This cannot happen unless the paths of the balls are identical.
 - (d) **This will always happen - No additional condition is required.**
19. A battery is connected by wires to a bulb as shown below and the bulb glows. Through which points does the charge flow?



- (a) **1-2-3-4-1. Charge flows through the battery also.**
- (b) 1-2-3-4. Charge flows through the wires and bulb only.
- (c) 2-3. Charge flows only through the bulb.

- (d) There is no flow of the charge in the circuit.
20. Which of the following is true?
- (a) $\cos 80^\circ = -\sin 10^\circ$
 - (b) $\cos 120^\circ = -\cos 240^\circ$
 - (c) $\sin 135^\circ = -\sin 270^\circ$
 - (d) $\sin 330^\circ = \sin 210^\circ$
- The answer is **(d)**.

AB
CSE

Part B: Analytical Questions

21. A year in Solar calendar consist of 365.25 days and the same in Lunar calendar consist of 354 days. The additional days in Solar calendar are kept as balance every year. Whenever the number of balance days exceeds 30, an additional month of 30 days is added to the lunar year to offset the difference. The cycle goes on. Anwesh, whose birthday falls on 1st January, noticed that in the year 2008, his birthday coincided with the start of the lunar year. In which earliest future year, his birthday will again coincide with the start of the lunar year?

Solution: Every year the Solar year lags by 11.25 days. After 4 years, number of intercalary day will be integer, i.e. 45.

Intercalary days are compensated by a month whenever they exceed 30 days.

Thus, one has to find L.C.M. of 45 and 30.

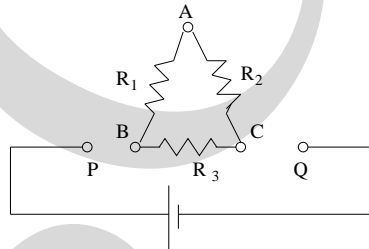
L.C.M. is 90. i.e. after 90 intercalary days are introduced, both calenders will match.

i.e. they will match after 8 years.

Thus, his birthday in **2016** will again mark start of the lunar calander.

Important: Brute force method should not be given more than 7 marks.

22. Find R_1 , R_2 and R_3 in the circuit diagram, given the conditions below:



- If P is connected to A & Q is connected to C then the current in the circuit is 2A.
- If P is connected to A & Q is connected to B then the current in the circuit is 4A.
- If P is connected to C & Q is connected to B then the current in the circuit is 3A.

Solution: For a given V,

$$V = 2 \frac{(R_1 + R_3) R_2}{(R_1 + R_2 + R_3)} \quad (1)$$

$$= 4 \frac{(R_2 + R_3) R_1}{(R_1 + R_2 + R_3)} \quad (2)$$

$$= 3 \frac{(R_1 + R_2) R_3}{(R_1 + R_2 + R_3)} \quad (3)$$

Thus, (4)

$$4(R_2 + R_3) R_1 = 2(R_1 + R_3) R_2 \quad (5)$$

$$2R_1 R_2 = 2R_2 R_3 - 4R_1 R_3$$

$$R_1 R_2 = R_2 R_3 - 2R_1 R_3 \quad (6)$$

$$4(R_2 + R_3) R_1 = 3(R_1 + R_2) R_3 \quad (7)$$

$$R_1 R_3 = 3R_2 R_3 - 4R_1 R_2 \quad (8)$$

$$R_1 R_3 = 3R_2 R_3 - 4R_2 R_3 + 8R_1 R_3$$

$$R_2 R_3 = 7R_1 R_3$$

$$R_2 = 7R_1 \quad (9)$$

$$2(R_1 + R_3) R_2 = 3(R_1 + R_2) R_3 \quad (10)$$

$$R_2 R_3 = 2R_1 R_2 - 3R_1 R_3 \quad (11)$$

$$7R_1 R_3 = 14R_1^2 - 3R_1 R_3$$

$$10R_1 R_3 = 14R_1^2$$

$$R_3 = \frac{7}{5} R_1 \quad (12)$$

The resistances are in ratio $\mathbf{R}_1 : \mathbf{R}_2 : \mathbf{R}_3 = 1 : 7 : \frac{7}{5}$ (13)

$$(R_1 + R_2 + R_3) = \frac{47}{5} R_1 \quad (14)$$

$$R_3 = \frac{V(R_1 + R_2 + R_3)}{3(R_1 + R_2)} = \frac{\frac{47}{5} V R_1}{3 \times 8R_1}$$

$$\mathbf{R}_3 = \frac{47}{120} V \quad (15)$$

$$R_1 = \frac{5}{7} \times \frac{47}{120} V$$

$$\mathbf{R}_1 = \frac{47}{168} V \quad (16)$$

$$R_2 = 7 \times \frac{47}{168} V$$

$$\mathbf{R}_2 = \frac{47}{24} V \quad (17)$$

Note: If one finds ratio of resistance correctly, 8.5 marks out of 10 should be awarded.

23. You are given two lenses of focal lengths f_1 and f_2 respectively.
- Is it possible to arrange them in such a way that both incident beam and emergent beam of light will be parallel beams? Show all (at least 3) possible cases (for different lens combinations) with ray diagrams.
 - Are the incident and emergent beams parallel to each other?
 - By observing the ray diagrams, state the condition on the distance 'd' between the two lenses in terms of f_1 and f_2 .
 - For what combinations of lenses, the said arrangement is not possible?

Solution:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

d is the distance between the two lenses.

If the incident beam as well as the emerging beam are parallel beams, then $\frac{1}{f} = 0$ which gives the condition, $d = f_1 + f_2$

If one lens is concave and other convex, it will become, $d = f_1 - f_2$

Thus, it is only possible if focal length of the convex lens is more than that of the concave lens.

If both lenses are convex, d is always positive. Hence it is always possible.

If both lenses are concave, it is not possible at all.

Note: The ray diagrams should be such that above relations could be inferred by measuring respective lengths on the ray diagrams.

The two beams will be parallel to each other if both lenses are parallel to each other. Knowledge of the first equation is not expected. Students should infer the $d = f_1 + f_2$ relation by purely observing ray diagrams.

24. If Aniket starts drawing a straight line with a brand new typical ball-point pen, how long line can be drawn before he finishes his refill? Explicitly state all the assumptions you make.

Solution: A typical ball pen refill has length of 12 cm.

(Acceptable 10-15 cm)

Typical refill diameter is 1 mm.

(Acceptable 0.5 - 2 mm)

Hence Total volume of ink is $\pi r^2 h = \pi(0.05)^2 \times 12$ cc

$V \approx 0.1$ cc

Typical thickness of writing is of the size of finite number of molecules.

Size of one ink molecule can be taken to be 0.5 - 1nm.

Hence the thickness would be roughly 10nm.

(Acceptable 0.5-50 nm)

Typical width is half of refill diameter.

Thus, length,

$$l = \frac{V}{td} = \frac{\pi r^2 h}{2tr} = \frac{\pi \times 0.05 \times 12}{2 \times 10 \times 10^{-7}} \text{cm}$$

$l \approx 9.5$ km.

Note: Answer is not important in this order of magnitude estimation question. Approach to the problem should be judged for marks.