Indian National Astronomy Olympiad – 2015

Question Paper — INAO – 2015

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ — Date: 31st January 2015

Duration: **Three Hours** — Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 3 pages (5 sides).
- There are total 9 questions. Maximum marks are indicated in front of each question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- No computational aides like calculators, log tables, slide rule etc. are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative dates for the next stage are as follows:
  - Orientation Camp (Junior): 29th April to 7th May 2015.
  - Orientation Camp (Senior): 2nd May to 7th May 2015.
  - Selection Camp (Jr. + Sr.): 26th May to 5th June 2015.
  - Participation in both parts (Orientation and Selection) is mandatory for all participants.

Useful Physical Constants

Mass of the Earth	M_E	$\approx$	6.0×10^{24} kg
Radius of the Earth	R_E	$\approx$	6.4×10^{6} m
Mass of Jupiter	M_J	$\approx$	2.0×10^{27} kg
Speed of Light	c	$\approx$	3.0×10^{8} m/s
Astronomical Unit	1 A. U.	$\approx$	1.5×10^{11} m
Solar Luminosity	$L_\odot$	$\approx$	3.8×10^{26} W
Gravitational Constant	G	$\approx$	$\frac{20}{3} \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Avogadro constant	N_a	$\approx$	6.0×10^{23} mol^{-1}
Charge of an electron	e	$\approx$	1.6×10^{-19} C
Permeability of free space	μ_0	$\approx$	$4\pi \times 10^{-7}$ N A^{-2}
Stephan's Constant	σ	$=$	$\frac{17}{3} \times 10^{-8}\ Wm^{-2}K^{-4}$

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1. In science fiction novel “The Evitable Conflict”, Isaac Asimov suggested that, in a futuristic world, all food items may be replaced with a nutritious powder made of wheat flour. What size of world population can be sustained with this food staple? You may assume that the average annual yield of wheat to be about 3.6 tonne per hectare (present day actual figure) and the geography of the Earth to remain similar to the present world. (10)

Solution:

Surface Area of the Earth = 4π r^2 = 5.2×10^{14} m^2
Only 30% of it is land. Thus, total land mass is about 1.5×10^{14} m^2.
By rough estimate, ignoring mountains, deserts, arid regions, forests, towns / cities etc., only 10% of the land is cultivable.
Thus, cultivable land is only 1.5×10^{13} m^2 i.e. 1.5 billion hectare.
If all this land is used to cultivate wheat then, annual total wheat production will be $1.5 \times 10^9 \times 3600 = 5.4$ trillion kg.
Now each person on average needs 2000-3000 calories per day, which may be obtained by less than half kg of wheat flour. One can estimate daily intake in different ways but the number obtained would be roughly similar. e.g. One can argue that a typical human eats about 250 gm of food per meal and you need 3 meals per day. Thus total is 750 gm. However, water content in all food items is more than half. Thus, actual weight of food is only half of that. Or one can argue each roti needs about 25 gm of flour and if you are not eating anything else, then one may end up eating 15-20 rotis per day.
Thus, each person would need about 400 gm wheat flour per day. i.e. about 145 kg per year.
i.e. sustainable population would be $5.4 \times 10^{12}/145 = 40$ billion people.

Note: As with all estimation questions, the arguments presented and validity of assumptions is more important than the exact numbers.

2. Match the following (each entry in column A may map with **zero** or **one** or **more** entries in column B but each entry in column B has **exactly one** matching entry in column A). (10)

Column A	**Column B**
Satellite	Whirlpool
Galaxy	Sirius
Star	Ceres
Star cluster (group)	Pegasus
Dwarf Planet	Pallas
Asteroid	Puppis
Constellation	Pluto
Planet	Phobos
	Pleiades
	Pollux

Solution:

Satellite - Phobos	Galaxy - Whirlpool
Star - Sirius, Pollux	Cluster - Pleiades
Dwarf Planet - Pluto, Ceres	Asteroid - Pallas
Constellation - Pegasus, Puppis	Planet -

3. Two identical satellites A and B are launched in equatorial circular orbits of period 4.8 hours. Satellite A rotates in the sense of rotation of the Earth while satellite B rotates in the opposite sense. The orbits are separated slightly to avoid collision. On a particular day, at 12 noon, both the satellites were seen exactly overhead by an observer on the Earth's equator. What is the minimum duration of time after which both these satellites will again be seen exactly overhead from the same place? (6)

Solution:

As the earth's rotation period is 24 hours, we have to find apparent time period of these two satellites with respect to an observer on the Earth.

$$\begin{aligned}
\omega_A &= \omega_0 - \omega_E \\
\therefore \frac{1}{T_A} &= \frac{1}{T_0} - \frac{1}{T_E} = \frac{1}{4.8} - \frac{1}{24} \\
T_A &= \frac{24}{(5-1)} = 6\,\text{hrs} \\
\omega_B &= \omega_0 + \omega_E \\
\therefore T_B &= \frac{24}{(5+1)} = 4\,\text{hrs}
\end{aligned}$$

L.C.M. of the two apparent periods is 12 hours. Thus, the two satellites will be seen overhead together after 12 hours.

Note: Due to printing issues, some students read the value of satellite orbital period as 1.8 hours. Answer in that case would be 72 hours. This case will also get full credit.

4. For each of the following statements, state if the statement is true or false **and** give a single line justification for your answer in each case. (15)

   1. All stars except pole star rise in the east and set in the west.
   2. The Earth's axis is changing its direction slowly because its magnetic pole is not properly aligned with the geographic pole.
   3. Cosmic rays are not part of electromagnetic spectrum.
   4. Some sunspots can be bigger than the Earth.
   5. Jupiter can fit in the space between the Earth's surface and the Moon's orbit.

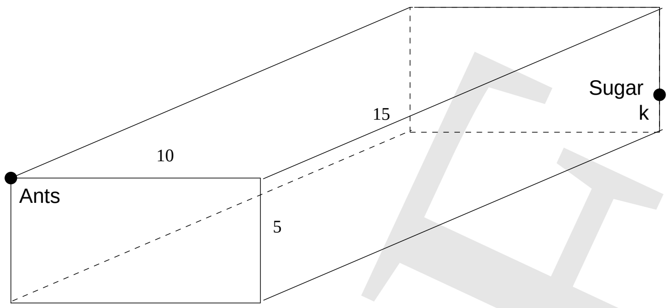
6. That the Earth does not fall into the Sun is a direct consequence of the Newton's third Law of motion.
7. The time taken by Mars Orbiter Mission (Mangalyaan) from the moment it left the Earth's orbit till it was captured by the gravity of Mars, is well determined by the Kepler's laws.
8. On a new moon day, the Earth's shadow covers the moon fully.
9. The stars which appear brighter are also closer to the Sun.
10. Pole star is visible from all the locations from the Earth.

Solution:

1. False. As an example, several stars around pole star would never go below horizon.
2. False. Although the Earth's axis is indeed precessing, this is not related to magnetic field of the Earth.
3. True. Cosmic rays is a term used for particle showers.
4. True. The Sun's radius is 100 times bigger than the Earth and many times we see big sunspots on the solar disk. A sunspot which is barely of the size of the Earth will not be visible through small telescope.
5. True. Jupiter is just 10-15 times bigger than the Earth whereas lunar orbit is 60 times bigger.
6. False. Action and reaction forces act on different bodies and hence reaction force cannot be seen as the one balancing the gravitational force.
7. True. From the time it left the Earth's orbit till the time it was captured by Martian gravity, it's trajectory was part of an effective orbit around the Sun.
8. False. On a new moon day, the moon is between the Sun and the Earth. So the Earth's shadow cannot fall on the Moon.
9. False. The visible brightness depends on distance of the star as well as its intrinsic brightness.
10. False. Polaris is not visible from southern hemisphere.

5. A big hall is 15 meter long, 10 meter wide and 5 meter high. Two ants are seated inside this hall at one of the upper corners. On the edge, diagonally opposite to the ants, there is a sugar cube at a height k meters above the ground (see diagram). One of the ants decides to go down the vertical edge, across the floor diagonal, and then up the opposite vertical edge to reach the sugar cube. The second ant tries to take shortest route along the walls without touching either the floor or the ceiling. Both (7)

the ants move with the same uniform speed and you can assume that no time is spent in changing the direction at any point. Find condition on k such that the first ant reaches the sugar cube first.

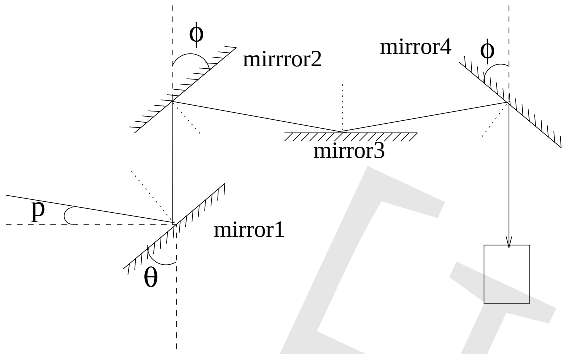


Solution:

The path length for the first ant $= h + \sqrt{l^2 + w^2} + k$
The path length for the second ant $= \sqrt{(l+w)^2 + (h-k)^2}$
Our condition is,

$$
\begin{aligned}
h + \sqrt{l^2 + w^2} + k &< \sqrt{(l+w)^2 + (h-k)^2} \\
\therefore 5 + \sqrt{15^2 + 10^2} + k &< \sqrt{(15+10)^2 + (5-k)^2} \\
(5+k) + \sqrt{325} &< \sqrt{625 + (5-k)^2} \\
\text{squaring, } (5+k)^2 + 325 + 2\sqrt{325}(5+k) &< 625 + (5-k)^2 \\
\cancel{25} + 10k + \cancel{k^2} + 325 + 10\sqrt{13}(5+k) &< 625 + \cancel{25} - 10k + \cancel{k^2} \\
10\sqrt{13}(5+k) + 20k &< 300 \\
5\sqrt{13} + (2+\sqrt{13})k &< 30 \\
k &< \frac{5 \times (6-\sqrt{13})}{(2+\sqrt{13})} \\
\text{approximately, } k &< 2
\end{aligned}
$$

6. A ray of light enters an assembly of plane mirrors (from left) as shown in the figure below and undergoes reflection at all the four mirrors. After the last reflection, the ray travels in vertical direction and enters in the detecting instrument. In the figure, the dotted lines are normals for respective mirrors and dashed lines are exactly vertical or horizontal. The third mirror is exactly horizontal. It is known that angle θ is $50°$. Angle ϕ is unknown. Find angle p. (6)



Solution:
Let angle of incidences at the four mirrors be i_1, i_2, i_3 and i_4 respectively.

At mirror 1,

$$(90^\circ - \theta) + p = 90^\circ - i_1$$
$$\therefore i_1 = \theta - p$$

For ray from mirror 1 to mirror 2,

$$90^\circ + \theta + i_1 + 90^\circ - i_2 + 180^\circ - \phi = 360^\circ$$
$$\therefore \theta + i_1 = i_2 + \phi$$
$$\therefore i_2 = 2\theta - p - \phi$$

For ray from mirror 2 to mirror 3,

$$90^\circ - i_2 + \phi + i_3 = 180^\circ$$
$$\therefore \phi - i_2 + i_3 = 90^\circ$$
$$\phi - (2\theta - p - \phi) + i_3 = 90^\circ$$
$$\therefore i_3 = 90^\circ + 2\theta - p - 2\phi$$

For ray from mirror 3 to mirror 4,

$$i_3 = 2i_4$$
$$\therefore 2i_4 = 90^\circ + 2\theta - 2\phi - p$$

At mirror 4,

$$90^\circ + i_4 + \phi = 180^\circ$$
$$\therefore i_4 + \phi = 90^\circ$$
$$2i_4 + 2\phi = 180^\circ$$
$$90^\circ + 2\theta - 2\phi - p + 2\phi = 180^\circ$$
$$\therefore p = 2\theta - 90^\circ$$
$$= 2 \times 50^\circ - 90^\circ$$
$$\therefore p = 10^\circ$$

7. A spaceship sends a proton beam with cross-sectional area of $100\,\mathrm{m}^2$ at a speed of $0.01\,c$ in the plane of the Earth's magnetic equator. The closest distance of the beam to the center of the earth is $1.28 \times 10^5\,\mathrm{km}$. The magnetic field at the midpoint of the distance separating the proton beam and the center of the earth is zero.
   (a) Find the number of protons emitted by the spaceship per second. (8)
   (b) As seen by the alien controlling the proton beam from the spaceship, would the Earth be on the left side or the right side? (3)
   (c) If mass of the spaceship is 1667 tonne, how much will be its recoil velocity due to this proton beam? (3)

   Note: The strength of the Earth's magnetic field as measured on the Earth's surface at magnetic equator is about 5×10^{-5} Tesla.

Solution:

Let $2r_0$ the shortest distance between the proton beam and the Earth. Let B_E be strength of the magnetic field at the Earth's surface (with effective pole strength M_E). Let B_{OE} be the magnetic field at r_0 due to earth's magnetic field and B_{OP} is the magnetic field due to I_p. As close to the Earth's surface, the Earth's magnetic field acts like a dipole,

$$
\begin{aligned}
B_E &= \frac{\mu_0\, M_E}{4\,\pi\, R_e^3} \\
\Rightarrow \frac{\mu_0\, M_E}{4\,\pi} &= R_E^3\, B_E \\
B_{OE} &= \frac{\mu_0\, M_E}{4\,\pi\, r_0^3} = \frac{R_E^3\, B_E}{r_0^3} \\
r_0 &= 10\, R_E \\
\therefore B_{OE} &= 0.001\, B_E \\
B_{OP} &= \frac{\mu_0\, I_P}{2\,\pi\, r_0} \\
\mid B_{OE} \mid &= \mid B_{OP} \mid \\
0.001\, B_E &= \frac{\mu_0\, I_P}{2\,\pi\, r_0} \\
\therefore I_P &= \frac{0.001\, B_E\, r_0}{2 \times \frac{\mu_0}{4\,\pi}} \\
&= \frac{0.001 \times 5 \times 10^{-5} \times 6.4 \times 10^7}{2 \times 10^{-7}} \\
\therefore I_P &= 1.6 \times 10^7\ \mathrm{A}
\end{aligned}
$$

Charge on a proton is same as that on an electron

$$
\begin{aligned}
I_P &= eN \\
N &= \frac{1.6 \times 10^7}{1.6 \times 10^{-19}} \\
&= 10^{26} \text{ protons per second}
\end{aligned}
$$

(b) The field lines from the Earth's magnetic field go from (geographic) southern hemisphere to northern hemisphere. To cancel this field, by virtue of right hand rule, the spaceship should be placed such that for an observer on the ship, the Earth would be seen on the right side.

Some students might take the alien to be upside down and get the answer as left side. If this case is presented clearly and with proper reasoning, credit will be given for the same.

(c) As protons are same as Hydrogen ions, one can say N_A protons would weigh about 1 g. Thus, for every second,

$$
\begin{aligned}
M_{ship} v_{ship} &= M_{beam} v_{beam} \\
v_{ship} &= \frac{1 \times 10^{-3}\, N \times 0.01\, c}{N_A\, M_{ship}} \\
&= \frac{1 \times 10^{-3} \times 1 \times 10^{26} \times 3 \times 10^{6}}{6 \times 10^{23} \times 1660 \times 10^{3}} \\
v_{ship} &= 0.3\,\mathrm{m/s}
\end{aligned}
$$

Total velocity of the ship after t seconds would be $0.3t$m/s.

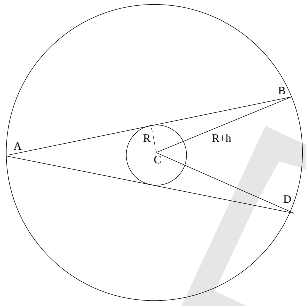
8. India plans to launch N communication satellites, all of which are to be in a geostationary orbit around the equator. All satellites are arranged equi-spaced in the same orbit. Find all the possible values of N such that each satellite can see all the other satellites of the series at any instant of time. (8)

Solution:

For geostationary orbits,

$$
\begin{aligned}
a^3 &= \frac{G M_E T_E^2}{4\pi^2} \\
&= \frac{\frac{20}{3} \times 10^{-11} \times 6 \times 10^{24} \times (86400)^2}{40} \\
&= 8.64^2 \times 10^{21} \\
\therefore a &\approx 4.2 \times 10^{7}\,\mathrm{m} \\
(R + h) &\approx 42\,000\,\mathrm{km}
\end{aligned}
$$

First of all, if the number of satellites are even then for every satellite, there will always be another satellite at diametrically opposite position. Clearly any satellite cannot see the diametrically opposite satellite. So n cannot be even.
If we draw tangents to the Earth's surface from any satellite (as shown in the figure), it becomes clear that minimum angle between the two consecutive satellites as seen from the Earth would be $\measuredangle BCD$.



$$\begin{aligned}\measuredangle BCD &= 2 \times \measuredangle BAD \\ &= 2 \times 2 \times \measuredangle ABC \\ &= 4 \sin^{-1}\left(\frac{R}{R+h}\right) \\ &\approx \frac{4 \times 6400}{42000} \\ &\approx \frac{4}{7} \text{ rad}\end{aligned}$$

As a circle has 2π radians, N can take following values:
$N \in \{3, 5, 7, 9, 11\}$
Note: Exact calculations show that N should be less than 11. Thus, 11 will be taken as marginal case. Solutions with or without 11 will get full credit.

9. The given graph is a scatter-plot of all exoplanets from the *Open Exoplanet Catalogue*, including those discovered by the *Kepler* mission. This *Kepler* mission is a special space based instrument, which has been continuously observing same small part of the sky from the start of its operation to measure any small changes in the brightness of the individual stars in that particular direction. The scales on the graph are as follows:
Top - log of orbital period of the planet in years
Right - log of mass of the planet as a multiple of mass of the Earth
Bottom - log of orbital radius of the planet
Left - log of mass of the planet as a multiple of mass of Jupiter

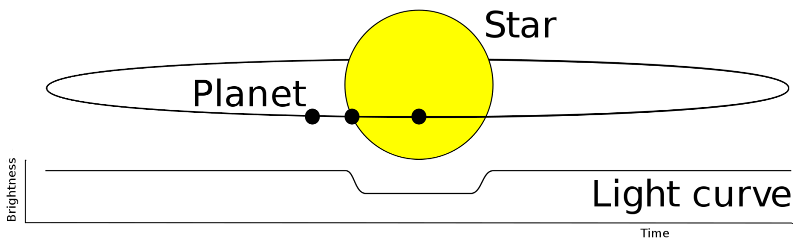
For example, on the left side, 0 would correspond to $\log_{10}(m/M_J) = 0$ i.e. $m = M_J$. As a way to simplify the problem, let us assume that all the parent stars of these exoplanets have exactly same mass as that of the Sun.

(a) Complete the scale for the $log_{10}P(years)$ and $log_{10}m(M_E)$ axes by providing values on the respective axes. (4)

(b) Mercury's distance from the Sun is about 40% that of the Earth and its mass is about 20 times smaller. Neptune's orbital radius is 30 times larger than the Earth's orbital radius and Neptune is 17 times more massive than the Earth. Mark the approximate positions of Mercury and Neptune on the graph. (4)

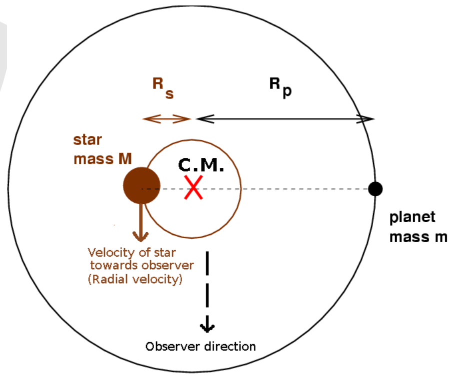
We further assume that all the parent stars emit same total energy per second (called Luminosity) as the Sun and all exoplanet orbits are circular.

(c) If a planet can sustain liquid water on its surface, it is said to be in the "habitable zone" around the parent star. Mark approximate edges of the habitable zone on the graph. State clearly the assumptions made in the process. (5)



One way of detecting exoplanets is called the *Transit method* (green dots in the graph). If a planet crosses (transits) in front of its parent star's disk as seen from the earth, then the observed visual brightness of the star drops by a small amount because the planet blocks some of the light. Most of the exoplanets detected by this method were discovered by the *Kepler* mission, which will complete it's sixth year of operation in a few months time. To confirm a detection, it is necessary that at least three transits of the given exoplanet should have been observed by the *Kepler* mission.

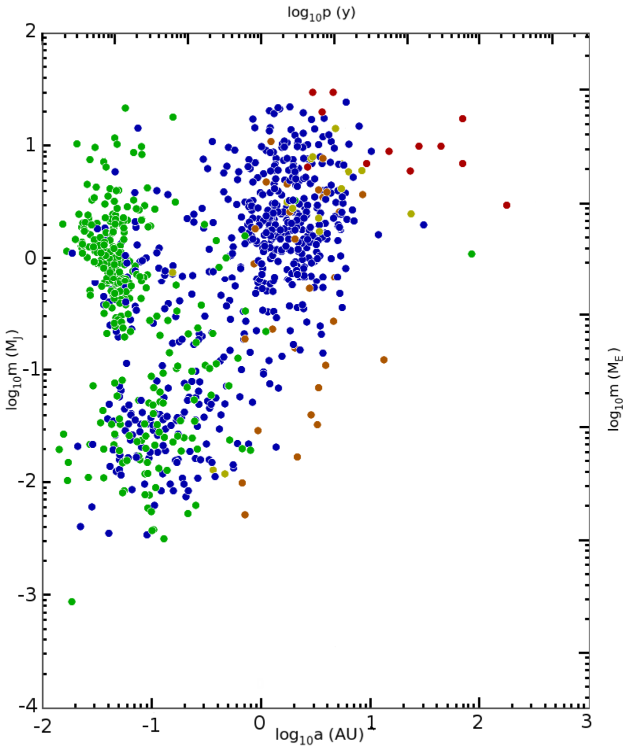
(d) Assume that the average density of the exoplanet is almost the same as that of the host star. It is known that the *Kepler* mission can detect about 0.01% variation in the light coming from the star. Using this information, draw two straight lines indicating detection limit for the transit method by Kepler spacecraft. Give brief justification of your answer. (5)



Another way is called the *Radial Velocity method* (blue dots). Here, we note that the host star and the exoplanet are both orbiting around the common centre of mass (C.M.) and hence the host star shows a periodic variation in its velocity towards or away from the observer (us). As the planet is not emitting any light, we just measure the velocity of the host star using the Doppler effect. This orbital velocity of the host star around the common centre of mass can help us determine mass of the exoplanet.

(e) Suppose the smallest change in radial velocity possible to detect by a certain spectrograph is $\sqrt{8}$ m/s. If we draw a straight line on the graph showing this limit, what would be the slope of this line? Draw this line on the graph. Looking at the exoplanets plotted on the graph, can we conclude that there exist some other instruments that could detect smaller changes in the radial velocity? Justify. (6)

Note: Although these two methods account for most of the exoplanets discovered, few of the exoplanets (also shown in this plot) are discovered by other methods like direct imaging, microlensing and timing analysis.


log10p (y)
2
1
0
-1
-2
-3
-4
log10m (MJ)
log10m (ME)
-2
-1
0
1
2
3
log10a (AU)

Solution:

(a) The orbital radius of the Earth is 1 AU and its orbital period is 1 year. Thus, for the Earth,

$$\log_{10} a(AU) = \log_{10} p(year) = 0$$

Using this one can put marks on the top scale. Note that the markings are closer than bottom scale as $p^2 \propto a^3$.
For right side scale, we note that $\frac{m_J}{m_E} = 300$. Thus, the mark approximately at the same level as 0 on the left scale corresponds to ratio of 300. Using this we make marks on the right scale.

(b) Use the orbital radius and mass ratio values to put these planets at appropriate places.

(c) Let us assume the planet is in thermal equilibrium. Thus, amount of heat absorbed would be same as the amount heat emitted.

$$\frac{L_\odot \pi r^2}{4\pi a^2} = 4\pi r^2 \sigma T^4$$
$$\therefore a = \frac{1}{4T^2}\sqrt{\frac{L_\odot}{\pi\sigma}}$$

We notice that this equation is independent of planet parameters. Thus, the boundaries will be vertical lines. At inner boundary, the equilibrium temperature should be 273 K and at outer boundary it should be 373 K.

$$\begin{aligned}
a_{out} &= \frac{1}{4\times(270)^2}\sqrt{\frac{4\times10^{26}}{\frac{22}{7}\times\frac{17}{3}\times10^{-8}}} \\
&= \frac{1\times10^{17}}{4\times7.3\times10^4\times1.5\times10^{11}}\sqrt{\frac{4}{17}} \\
&= \frac{100}{2\times4\times7.3\times1.5} \approx 1.1 \text{ A. U.} \\
a_{in} &= \frac{100}{2\times4\times13.7\times1.5} \approx 0.6 \text{ A. U.}
\end{aligned}$$

Only approximate estimation is expected here.

(d) As the observer (us) is very very far from the star, the physical separation between planet and the star is negligible and the drop in intensity of star light during a planet transit depends only on the ratio of cross-sectional area. Thus, one should draw a horizontal line.

$$\begin{aligned}
\frac{A_{pl}}{A_{st}} &= \left(\frac{R_{pl}}{R_{st}}\right)^2 \\
&= \left(\frac{V_{pl}}{V_{st}}\right)^{\frac{2}{3}} \\
0.0001 &= \left(\frac{M_{pl}}{M_{st}}\right)^{\frac{2}{3}} \\
0.0001^{\frac{3}{2}} &= \left(\frac{M_{pl}}{M_{st}}\right) \\
M_{pl} &= 1 \times 10^{-6} M_{st} = 0.001\, M_J
\end{aligned}$$

Thus, one should draw horizontal line where $\log_{10} m(M_J) = -3$. Further, as mentioned in the beginning, Kepler has been in operation for nearly 6 years and it requires at least 3 transits to flag a detection. Thus, typical orbital period for exoplanets discovered by Kepler should be 2 years or smaller. Draw corresponding vertical line.

(e) As the orbit is circular, $v_{st} = a_{st}\omega$ and moments about the centre of mass are balanced.

$$\begin{aligned}
m_{st}a_{st} &= m_{pl}a_{pl} \\
v_{st} &= a_{st}\omega = \frac{m_{pl}a_{pl}}{m_{st}}\omega \\
&= \frac{m_{pl}a_{pl}}{m_{st}} \times \frac{2\pi}{T} \\
v_{st}^2 &= \frac{m_{pl}^2 a_{pl}^2}{m_{st}} \times \frac{4\pi^2}{T^2 m_{st}}
\end{aligned}$$

by Kepler's third law,

$$\begin{aligned}
v_{st}^2 &\approx \frac{m_{pl}^2 a_{pl}^2}{m_{st}} \times \frac{G}{a_{pl}^3} \\
&= \frac{G}{m_{st}} \times \frac{m_{pl}^2}{a_{pl}}
\end{aligned}$$

Thus, for any given velocity detection limit,

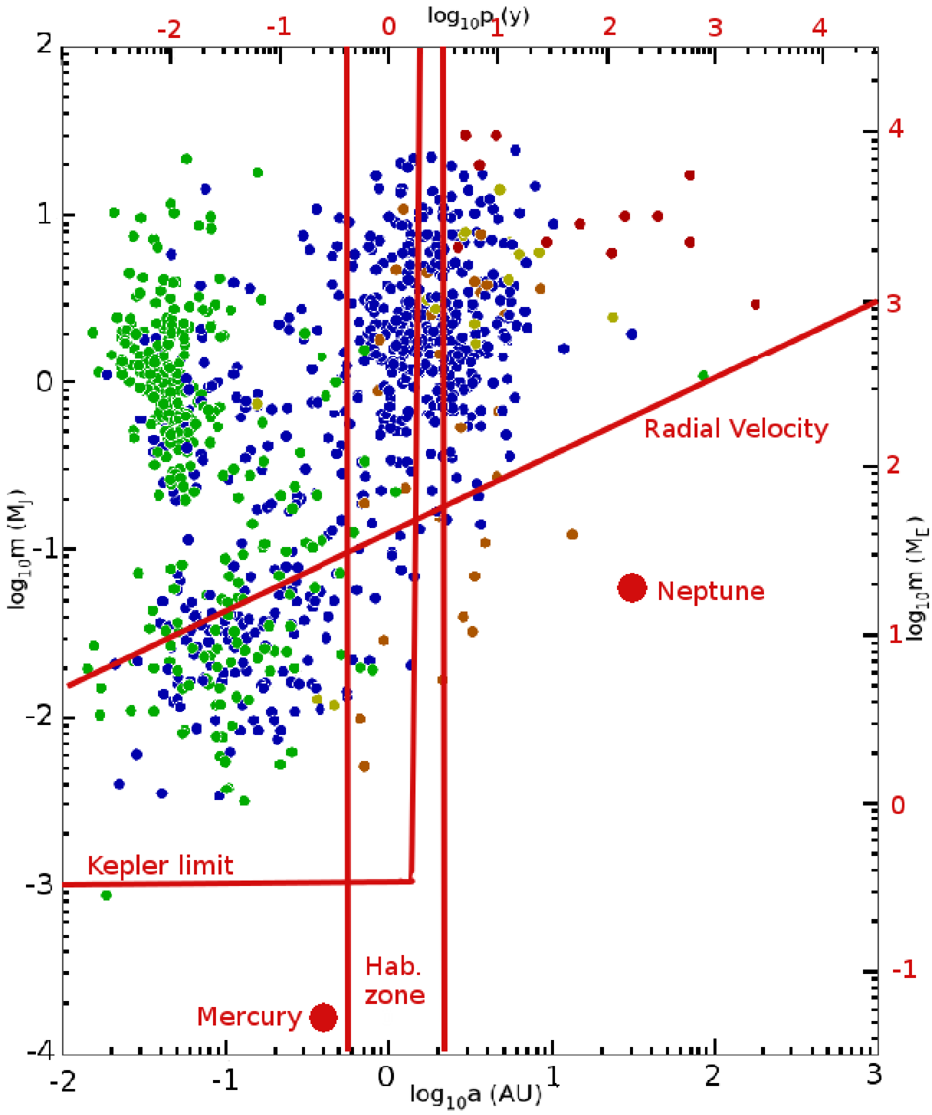
$$\begin{aligned}
a_{pl} &= \frac{G}{m_{st}v_{st}^2} m_{pl}^2 \\
a_{pl}(\text{in A. U.}) &= \frac{GM_E^2}{(1\text{ A. U.})m_{st}v_{st}^2} \times m_{pl}^2\ (\text{in } M_E^2) \\
\therefore [\log_{10} m_{pl}] &= \frac{1}{2}[\log_{10} a_{pl}] - \left[\frac{1}{2}\log_{10}\left(\frac{GM_E^2}{(1\text{ A. U.})m_{st}v_{st}^2}\right)\right]
\end{aligned}$$

$$= \frac{1}{2}\left[\log_{10} a_{pl}\right] - \left[\frac{1}{2}\log_{10}\left(\frac{20 \times 10^{-11} \times (6 \times 10^{24})^2}{3 \times 1.5 \times 10^{11} \times 2 \times 10^{30} \times (\sqrt{8})^2}\right)\right]$$

$$= \frac{1}{2}\left[\log_{10} a_{pl}\right] - \left[\frac{1}{2}\log_{10}\left(10^{-3}\right)\right]$$

$$= \frac{1}{2}\left[\log_{10} a_{pl}\right] + 1.5$$

This is equation of a straight line in the form $y = mx + c$. Thus, on the given plot, we have to draw a straight line with slope 0.5. Further, points $(-2, 0.5)$ and $(3, 3)$ are on the line. Once we draw the line, we realise that a lot of blue points are below this line. Thus, present detection limit should be lower than this value.



Notes for Junior Group

- **Do not solve question 7(a).** Assume that the answer for question 7(a) is 10^{26} photons per second.
- Question 4 will have 20 marks.
- Question 7(b) and 7(c) will have 4 marks each.
- Question 9(c) will have 6 marks.

Notes for Junior Group

- **Do not solve question 7(a).** Assume that the answer for question 7(a) is 10^{26} photons per second.
- Question 4 will have 20 marks.
- Question 7(b) and 7(c) will have 4 marks each.
- Question 9(c) will have 6 marks.

Notes for Junior Group

- **Do not solve question 7(a).** Assume that the answer for question 7(a) is 10^{26} photons per second.
- Question 4 will have 20 marks.
- Question 7(b) and 7(c) will have 4 marks each.
- Question 9(c) will have 6 marks.

Notes for Junior Group

- **Do not solve question 7(a).** Assume that the answer for question 7(a) is 10^{26} photons per second.
- Question 4 will have 20 marks.
- Question 7(b) and 7(c) will have 4 marks each.
- Question 9(c) will have 6 marks.