1. Let \( ABC \) be an acute-angled triangle. The circle \( \Gamma \) with \( BC \) as diameter intersects \( AB \) and \( AC \) again at \( P \) and \( Q \), respectively. Determine \( \angle BAC \) given that the orthocentre of triangle \( APQ \) lies on \( \Gamma \).

2. Let \( f(x) = x^3 + ax^2 + bx + c \) and \( g(x) = x^3 + bx^2 + cx + a \), where \( a, b, c \) are integers with \( c \neq 0 \). Suppose that the following conditions hold:

   (a) \( f(1) = 0 \);
   (b) the roots of \( g(x) = 0 \) are the squares of the roots of \( f(x) = 0 \).

   Find the value of \( a^{2013} + b^{2013} + c^{2013} \).

3. Find all primes \( p \) and \( q \) such that \( p \) divides \( q^2 - 4 \) and \( q \) divides \( p^2 - 1 \).

4. Find the number of 10-tuples \( (a_1, a_2, \ldots, a_{10}) \) of integers such that \( |a_1| \leq 1 \) and

\[
a_1^2 + a_2^2 + a_3^2 + \cdots + a_{10}^2 - a_1a_2 - a_2a_3 - a_3a_4 - \cdots - a_9a_{10} - a_{10}a_1 = 2.
\]

5. Let \( ABC \) be a triangle with \( \angle A = 90^\circ \) and \( AB = AC \). Let \( D \) and \( E \) be points on the segment \( BC \) such that \( BD : DE : EC = 3 : 5 : 4 \). Prove that \( \angle DAE = 45^\circ \).

6. Suppose that \( m \) and \( n \) are integers such that both the quadratic equations \( x^2 + mx - n = 0 \) and \( x^2 - mx + n = 0 \) have integer roots. Prove that \( n \) is divisible by 6.

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