

Regional Mathematical Olympiad-2014

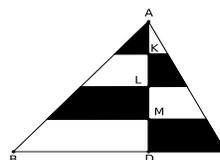
Time: 3 hours

December 07, 2014

Instructions:

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- Answer all the questions.
- All questions carry equal marks. Maximum marks: 102.
- Answer to each question should start on a new page. Clearly indicate the question number.

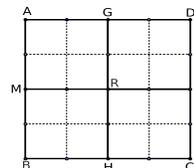
1. Let ABC be a triangle and let AD be the perpendicular from A on to BC . Let K, L, M be points on AD such that $AK = KL = LM = MD$. If the sum of the areas of the shaded regions is equal to the sum of the areas of the unshaded regions, prove that $BD = DC$.



2. Let a_1, a_2, \dots, a_{2n} be an arithmetic progression of positive real numbers with common difference d . Let
 (i) $a_1^2 + a_3^2 + \dots + a_{2n-1}^2 = x$, (ii) $a_2^2 + a_4^2 + \dots + a_{2n}^2 = y$, and
 (iii) $a_n + a_{n+1} = z$.
 Express d in terms of x, y, z, n .

3. Suppose for some positive integers r and s , the digits of 2^r is obtained by permuting the digits of 2^s in decimal expansion. Prove that $r = s$.

4. Is it possible to write the numbers $17, 18, 19, \dots, 32$ in a 4×4 grid of unit squares, with one number in each square, such that the product of the numbers in each 2×2 sub-grids $AMRG, GRND, MBHR$ and $RHCN$ is **divisible** by 16?



5. Let ABC be an acute-angled triangle and let H be its ortho-centre. For any point P on the circum-circle of triangle ABC , let Q be the point of intersection of the line BH with the line AP . Show that there is a unique point X on the circum-circle of ABC such that for every point $P \neq A, B$, the circum-circle of HQP pass through X .
6. Let $x_1, x_2, \dots, x_{2014}$ be positive real numbers such that $\sum_{j=1}^{2014} x_j = 1$. Determine with proof the smallest constant K such that

$$K \sum_{j=1}^{2014} \frac{x_j^2}{1 - x_j} \geq 1.$$