Sample Questions for PRMO 2017

1. Two positive integers $a$ and $b$ are such that $a + b = \frac{a}{b} + \frac{b}{a}$. What is the value of $a^2 + b^2$? [Ans: 02]

2. The equations $x^2 - 4x + k = 0$ and $x^2 + kx - 4 = 0$, where $k$ is a real number, have exactly one common root. What is the value of $k$? [Ans: 03]

3. Let $P(x)$ be a non-zero polynomial with integer coefficients. If $P(n)$ is divisible by $n$ for each positive integer $n$, what is the value of $P(0)$? [Ans: 00]

4. A natural number $k$ is such that $k^2 < 2014 < (k + 1)^2$. What is the largest prime factor of $k$? [Ans: 11]

5. How many two-digit positive integers $N$ have the property that the sum of $N$ and the number obtained reversing the order of the digits of $N$ is a perfect square? [Ans: 08]

6. What is the greatest possible perimeter of a right-angled triangle with integer side lengths if one of the sides has length 12? [Ans: 84]

7. In rectangle $ABCD$, $AB = 8$ and $BC = 20$. Let $P$ be a point on $AD$ such that $\angle BPC = 90^\circ$. If $r_1$, $r_2$, $r_3$ are the radii of the incircles of triangles $APB$, $BPC$ and $CPD$, what is the value of $r_1 + r_2 + r_3$? [Ans: 08]

8. Let $n$ be the largest integer that is the product of exactly 3 distinct prime numbers, $x$, $y$ and $10x + y$, where $x$ and $y$ are digits. What is the sum of the digits of $n$? [Ans: 12]

9. A subset $B$ of the set of first 100 positive integers has the property that no two elements of $B$ sum to 125. What is the maximum possible number of elements in $B$? [Ans: 62]

10. The circle $\omega$ touches the circle $\Omega$ internally at $P$. The centre $O$ of $\Omega$ is outside $\omega$. Let $XY$ be a diameter of $\Omega$ which is also tangent to $\omega$. Assume $PY > PX$. Let $PY$ intersect $\omega$ at $Z$. If $YZ = 2PZ$, what is the magnitude of $\angle PYX$ in degrees? [Ans: 15]