# Indian National Astronomy Olympiad – 2018

Question Paper INAO – 2018

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ Date: $27^{th}$ January 2018

Duration: **Three Hours** Maximum Marks: 100

*Please Note:*

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 4 pages (2 sheets).
- There are total 8 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Use of non-programmable scientific calculators is allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative dates for the next stage are as follows:
  - Orientation Cum Selection Camp (Senior): $24^{th}$ April to $11^{th}$ May 2018. This will be held at HBCSE, Mumbai.
  - Dates for IAO selection camp (Junior) will be announced by NCSM later.
  - Attending the camp for the entire duration is mandatory for all participants.

**Useful Constants**

| | | | |
|---|---|---|---|
| Mass of the Sun | $M_\odot$ | $\approx$ | $1.989 \times 10^{30}$ kg |
| Mass of the Earth | $M_\oplus$ | $\approx$ | $5.972 \times 10^{24}$ kg |
| Mass of the Moon | $M_{☾}$ | $\approx$ | $7.347 \times 10^{22}$ kg |
| Radius of the Earth | $R_\oplus$ | $\approx$ | $6.371 \times 10^{6}$ m |
| Speed of Light | $c$ | $\approx$ | $2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |
| Radius of the Sun | $R_\odot$ | $\approx$ | $6.955 \times 10^{8}$ m |
| Radius of the Moon | $R_m$ | $\approx$ | $1.737 \times 10^{6}$ m |
| Astronomical Unit | 1 A. U. | $\approx$ | $1.496 \times 10^{11}$ m |
| Solar Constant (at Earth) | S | $\approx$ | $1366\,\mathrm{W\,m^{-2}}$ |
| Gravitational Constant | $G$ | $\approx$ | $6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ |

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

1. (5 marks) In Newtonian mechanics force between two objects is felt instantaneously irrespective of distance. However, according to relativity, no interaction is truly instantaneous. For an object to feel the force, the information of the force is carried by field particles. But nothing can travel faster than the speed of light.
Two friends, 'A' (a follower of Newton) and 'B' (a follower of Einstein), were once debating about what will happen to the Earth if the Sun suddenly vanishes. They both calculated the direction in which the Earth will go, when this calamity happens. What will be the angular difference in the directions predicted by them?

**Solution:**

According to A, the force effect is instantaneous.
So, as soon as Sun disappears, Earth flies off tangentially. 1 M
According to B, the effect of disappearance of the Sun is felt with delay of 499 s. 1 M
In this time, the Earth would have moved by angle of

$$\frac{499 \times 360}{86400 \times 365.25} = 0.0056^\circ = 20.49'' = 9.77 \times 10^{-5}\,\text{rad}$$

Thus, the angular difference in the direction calculated by both of them will be 20.49″ 3 M

2. Most buses in India run on diesel, whose calorific value is $44.8 \times 10^6\,\text{J}\,\text{kg}^{-1}$ and density is $0.832\,\text{kg}\,\text{l}^{-1}$. A typical bus can go 3 km, on average, for every litre of diesel consumed.
   (a) (5 marks) Estimate power consumption of a typical bus.
   (b) (3 marks) Calculate the maximum amount of solar power incident on roof of a typical bus.
   (c) (2 marks) Best commercial solar cells in the market have an efficiency of 20%. If the bus is to be run purely on solar power (during the day), estimate the area of solar cells required. What is the ratio of this area to the area of the rooftop of the bus.

**Solution:**

Typical speed of bus in city = $18\,\text{km}\,\text{h}^{-1}$ to $36\,\text{km}\,\text{h}^{-1}$
Typical speed of bus on highway = $36\,\text{km}\,\text{h}^{-1}$ to $72\,\text{km}\,\text{h}^{-1}$
Considering efficiency of bus $3\,\text{km}\,\text{l}^{-1}$ and speed $30\,\text{km}\,\text{h}^{-1}$ 2 M
$\therefore$ Time to burn 1 litre of diesel is $t = \frac{3}{30} \times 3600 = 360\,\text{s}$ 1 M
$\therefore$ Power consumed by bus is, $P = \frac{44.80 \times 10^6 \times 0.832}{360} \approx 103.5\,\text{kW}$ 2 M
**Accepted range for power consumption of bus is** 75 kW **to** 300 kW, i.e. 100 hp to 200 hp
Typical rooftop area of bus in India $\approx 10\,\text{m} \times 2.5\,\text{m} = 25\,\text{m}^2$ (accepted range $10\,\text{m}^2$ to $30\,\text{m}^2$)
$\therefore$ Total solar energy received by bus $= 1.366 \times 10 \times 2.5 \approx 34.15\,\text{kW}$ 2 M
This assumes that all energy is incident perpendicularly on the rooftop, i.e. the Sun is

directly overhead 1 M
Considering efficiency of typical commercially available solar cell is 20%,
Total solar energy available for consumption is $0.2 \times 34.15\,\text{kW} = 6.83\,\text{kW}$. 1 M
$\therefore$ For minimum power consumption area of rooftop should be $\approx \dfrac{103.5}{6.83} \approx 15$ times more than that of present. 1 M

3. The star Kepler-13A has a Jupiter-like planet (Kepler-13Ab) revolving around it. The period of revolution and rotation for this planet are equal. Analysis of the planet's atmosphere has revealed that it contains vapourized Titanium Oxide (TiO), a key component in sunscreen lotions. Based on the boiling point of TiO, astronomers have estimated the temperature of Kepler-13Ab's upper atmosphere to be about 3000 K.
   (a) (7 marks) What is the orbital distance of Kepler-13Ab?
   Given: Surface temperature of the star Kepler-13A = 7560 K, Radius of the star Kepler-13A = 1.71 $R_\odot$
   (b) (2 marks) Further analysis reveals condensed TiO as part of the lower atmosphere in Kepler-13Ab. Can you think of an explanation for this phenomenon?

**Solution:**

Realize that the planet is tidally locked, which means only one hemisphere of the planet will receive radiation from the star at all times, and we have to consider the contribution to blackbody output of the planet only from this hemisphere.
Let that surface temperature of Kepler-13A, $T_s = 7560\,\text{K}$
Let radius of Kepler-13A, $R_s = 1.71\,R_\odot$
Using Stefan-Boltzmann's Law,

$$L_s = 4\pi\sigma R_s^2 T_s^4$$ 1 M

The flux reaching the planet is

$$F = \frac{4\pi\sigma R_s^2 T_s^4}{4\pi d^2}$$ 1 M

where $d$ is the orbital distance of Kepler-13Ab which we are interested in.
Now, flux absorbed by the hemisphere of planet is

$$E_{ab} = F(\pi R_p^2)$$ 2 M

where $R_p$ is the radius of the planet.
Energy emitted by the planet given its temperature $T_p$

$$E_{em} = 2\pi\sigma R_p^2 T_p^4$$ 1 M

Equating energy absorbed and emitted, we have

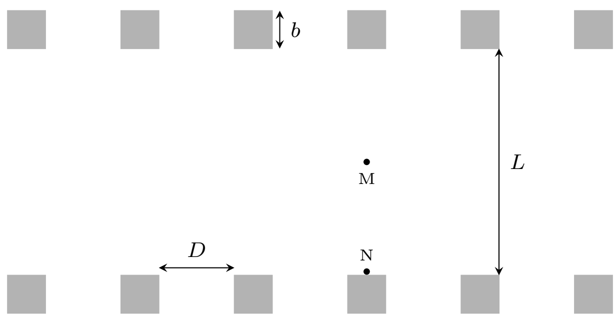
$$\begin{aligned} E_{ab} &= E_{em} \\ \frac{4\pi\sigma R_s^2 T_s^4}{4\pi d^2}\pi R_p^2 &= 2\pi\sigma R_p^2 T_p^4 \\ d &= \left(\frac{R_s^2 T_s^4}{2T_p^4}\right)^{\frac{1}{2}} \\ &= \frac{R_s T_s^2}{\sqrt{2}T_p^2} \\ &= \frac{1.71 R_\odot \times (7560)^2}{\sqrt{2}\times(3000)^2} \\ d &= 7.68\, R_\odot \approx 0.036\,\text{AU} \end{aligned}$$

1 M (for the step $d = \frac{R_s T_s^2}{\sqrt{2}T_p^2}$)

1 M (for $d = 7.68\, R_\odot \approx 0.036\,\text{AU}$)

For part (b), the ideal answer is: atmospheric currents transfer some vaporized TiO to the cold part of the planet where it condenses and travels back to the hot part of the planet at a lower height, where it rises again to re-vaporize. 2 M

4. Aditya is fascinated by the famous 'infinity corridor' of IITB. The corridor is a very long, straight, roofed pathway connecting different departments. The roof of the corridor is supported by square pillars of width $b$ on either side. Let $D$ be the distance between consecutive pillars on the same side and $L$ be the width of the pathway (see figure).

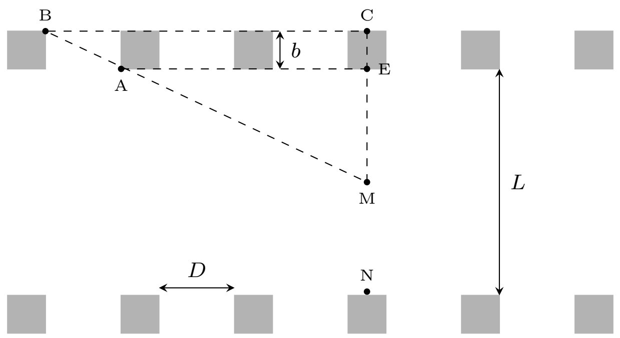



(a) (8 marks) One day Aditya was standing at point M (see figure), in the middle of the corridor, looking ahead along the path. He found that he cannot see any object outside the corridor beyond the $n^{\text{th}}$ pillar on his either side. Derive an expression for $n$.

(b) (3 marks) Assume that the space between the successive pillars is $D = 3\,\text{m}$, width of each pillar is $b = 0.3\,\text{m}$ and width of the corridor is $L = 5\,\text{m}$. Find numerical value of $n$.

(c) (4 marks) How will the answer in part (b) change, if Aditya was at position N instead of M (see figure) and looking through pillars along the top row of the figure?

**Solution:**




2 M

(a) A general condition can be stated as, 'nothing will be visible through the pillars, when the slope of ray $\vec{MA}$ is more than that of $\vec{MC}$'.

$$\frac{AE}{ME} \geq \frac{BC}{MC}$$

$$\therefore \frac{x}{L/2} \geq \frac{x+D}{L/2+b}$$ 2 M

$$\frac{xL}{2} + xb \geq \frac{xL}{2} + \frac{DL}{2}$$

$$\therefore x \geq \frac{DL}{2b}$$ 1 M

$$n = \frac{x - b/2}{D+b}$$ 1 M

$$\geq \frac{\frac{DL}{2b} - \frac{b}{2}}{D+b}$$

$$\therefore n \geq \frac{DL - b^2}{2b(D+b)}$$ 2 M

(b)

$$n \geq \frac{DL - b^2}{2b(D+b)}$$

$$\geq \frac{3 \times 5 - 0.3^2}{0.6 \times (3+0.3)}$$

$$n \geq 7.53$$

$$\therefore n = 8$$ 3 M

(c) Standing at the edge of the pathway,

$$\frac{AE}{NE} \geq \frac{BC}{NC}$$

$$\therefore \frac{x_e}{L} \geq \frac{x_e + D}{L+b}$$ 1 M

$$\therefore x_e \geq \frac{DL}{b}$$

$$n_e = \frac{x_e - b/2}{D+b}$$

$$n_e \geq \frac{2DL - b^2}{2b(D+b)}$$ 2 M

For given values, $n_e \geq 15.11$, i.e. $n_e = 16$ 1 M

5. Astronauts have found that the angular diameter of the Earth as seen from the surface of the Moon is 1.9°. We assume that:

- the observer is standing on the equator of the Earth.
- the Moon is in the equatorial plane.
- the Moon's orbit is circular.

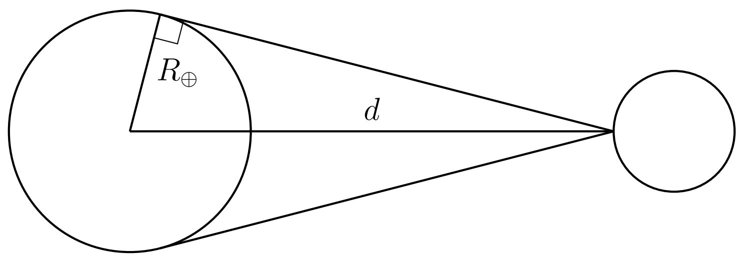
(a) (10 marks) Find the time it takes for the Moon to completely rise above the Earth's horizon.

(b) (3 marks) Briefly state why the three assumptions above are relevant to the solution.

(c) (2 marks) How much time would it take for the Earth to completely rise above Moon's horizon?

**Solution:**

(a) $\theta_\oplus = 1.9°$




1 M

$$\sin\left(\frac{\theta_\oplus}{2}\right) = \frac{R_\oplus}{d}$$

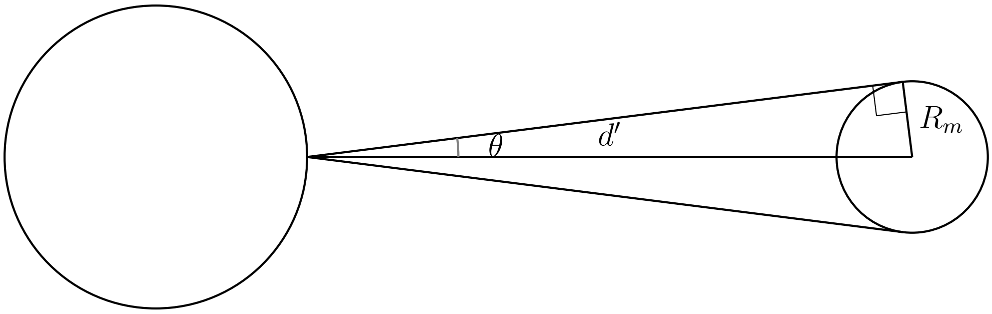
Since $\frac{\theta_\oplus}{2}$ is very small, $\sin\frac{\theta_\oplus}{2} \approx \frac{\theta_\oplus}{2}$.

$$\therefore \frac{\theta_\oplus}{2} = \frac{R_\oplus}{d}$$

$$\therefore d = \frac{2R_\oplus}{\theta_\oplus}$$

$$d = \frac{2 \times 6.371 \times 10^6}{1.9 \times \frac{\pi}{180}}$$

$$d = 3.842 \times 10^8 \text{ m}$$ 2 M




The angle subtended by Moon on Earth,

$$\theta_{\text{Moon}} = \frac{2 \times R_{☾}}{d'}$$

$$\text{But, } d' = d + R_{☾} - R_{\oplus}$$ 1 M

$$\therefore d' = 3.842 \times 10^8 + 1.737 \times 10^6 - 6.371 \times 10^6 = 3.796 \times 10^8$$

$$\therefore \theta_{\text{Moon}} = \frac{2 \times 1.737 \times 10^6}{3.796 \times 10^8} = 9.152 \times 10^{-3}\,\text{rad}$$ 1 M

$$\therefore \theta_{\text{Moon}} = 9.152 \times 10^{-3}\,\text{rad}$$

$$\text{But, } \frac{GM_{\oplus}m}{(d + R_{☾})^2} = m\omega^2 \times (d + R_{☾})$$

$\therefore$ Angular speed of Moon,

$$\omega_{\text{Moon}} = \sqrt{\frac{GM_{\oplus}}{(d + R_{☾})^3}}$$

$$\omega_{\text{Moon}} = \sqrt{\frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{(3.842 \times 10^8 + 1.737 \times 10^6)^3}} = \sqrt{\frac{39.86 \times 10^{13}}{(385.9 \times 10^6)^3}}$$

$$= 2.634 \times 10^{-6}\,\text{rad/s}$$ 2 M

$\therefore$Angular speed of Earth,

$$\omega_{\oplus} = \frac{2\pi}{T} = \frac{2\pi}{23.93 \times 3600} = 7.293 \times 10^{-5}\,\text{rad/s}$$ 1 M

Time to rise for the Moon as seen from the Earth,

$$t_{rise} = \frac{\theta_{\text{Moon}}}{(\omega_{\oplus} - \omega_{\text{Moon}})} = \frac{9.152 \times 10^{-3}}{\left(7.293 \times 10^{-5} - 0.263 \times 10^{-5}\right)} = 1.302 \times 10^2\,\text{s}$$ 2 M

$$t_{rise} = 130\,\text{s}$$

**If $\omega_{\text{Moon}}$ is not considered, and T is taken as 24 h then answer is 126 s. For this solution, penalty will be 3 Marks**

(b) • If the orbit of Moon is not circular, the $\omega_{Moon}$ will differ from point to point. 1 M

- If the observer was not on the equator, path of Moon in the sky will appear inclined and will change duration of rising. 1 M
- If the Moon is not in equatorial plane, the $\omega_{\text{Moon}}$ will have to corrected for this offset. 1 M

(c) Moon is tidally phase-locked with the Earth. So Earth does not rise or set on the surface of the Moon. 2 M

6. (a) (3 marks) Let us say we are observing sky from a dark location and all planets are visible in the sky. Arrange the planets of the solar system in the descending order of their apparent brightness as observed from the Earth.

   (b) (14 marks) Shinjini was observing the sky from a location on the equator on the night of 20-21 March and she made following observations in her diary.

   - Today is 11 days prior to the full Moon.
   - Saturn is seen in constellation of Sagittarius.
   - Jupiter is seen rising at the time of the Moon set.
   - Mars' position was coinciding with the centre of Milky Way.
   - Mercury set about 2 hours before the Moon.
   - Venus was seen in the evening sky for about 2 hours after sunset.

   In the answersheet, you will find a circle which is passing through East, Zenith (point exactly above the head of the observer), West and Nadir (point exactly below the observer). Use the information given above to mark positions of the Sun, the Moon and the 5 planets on this circle at 11 am on the Vernal Equinox day (21$^{\text{st}}$ March).
   For each object, write a 1-2 line explanation stating why you think it is the correct position of the object.

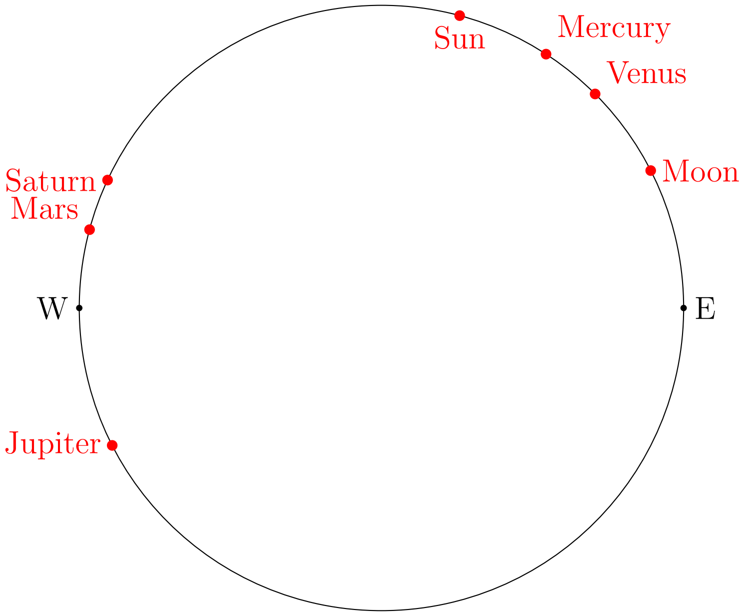
**Solution:**

(a) Venus, Jupiter, Saturn, Mars, Mercury, Uranus and Neptune.
No penalty if Mars and Mercury's positions in the list are interchanged.

(b) Positions of objects in sky at 11:00 on 21 March:

- As the time is 11:00 am, the Sun should be to the east of zenith by about 15°.
- The Moon moves by about 13° per day. So it will be 52° off from the new moon position. since it is waxing fortnight, the Moon should be seen in the sky at the Sunset. Thus, the Moon should be to the east of Sun. Taking into account the Sun's westward motion, the moon should be 48° east of the Sun.
- Today is Vernal Equinox day. Therefore, the Sun is in Pisces. If Saturn is in Sagittatius, it would be about 90° west of Sun.
- If Jupiter was seen rising at the time of Moon set, then it should be roughly opposite to the Moon.

- The centre of milky way is in Sagittarius. Hence the Mars should be at about the same position as the Saturn.
- If Mercury set about 2 hours before the Moon, then it is about 30° to the west of the Moon.
- If Venus set about 2 hours after the Sun, then it should be about 30° to the east of Sun.

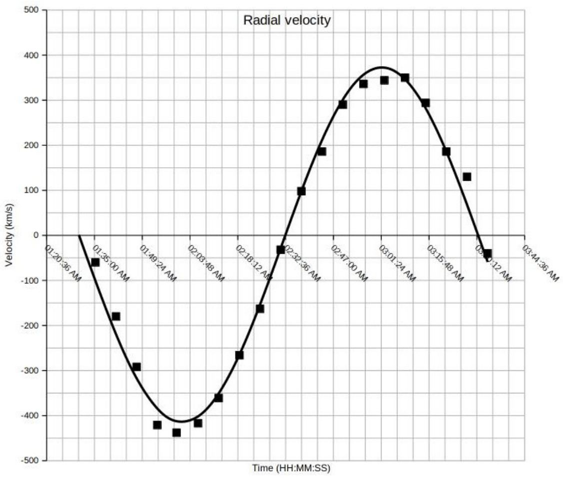



7. Yash discovered a binary star, consisting of a very light star A in a circular orbit around a massive star B ($m_A \ll m_B$). He could measure the component of velocity of star A along the line of sight via Doppler shift. His measurements are tabulated below. All the data were obtained on 17$^{\text{th}}$ July 2009, and the times given are in UT.
   (a) (10 marks) Plot the data, and measure the orbital period of star A from the plot.
   (b) (5 marks) Use this information to calculate the mass of star B, in terms of solar mass.
   (c) (3 marks) Is the plot symmetric with resepect to time axis? If yes, what does it signify? If no, what is the reason?

| Time (UT) | Velocity (km/s) | Time (UT) | Velocity (km/s) |
|---|---|---|---|
| 01:47:44 | -292 | 02:37:21 | 98 |
| 01:41:31 | -180 | 02:43:34 | 186 |
| 01:35:18 | -60 | 02:49:51 | 290 |
| 01:53:57 | -421 | 02:56:04 | 336 |
| 01:59:45 | -438 | 03:02:22 | 344 |
| 02:06:13 | -417 | 03:08:35 | 350 |
| 02:12:26 | -361 | 03:14:48 | 294 |
| 02:18:41 | -266 | 03:21:01 | 186 |
| 02:24:54 | -163 | 03:27:15 | 130 |
| 02:31:07 | -32 | 03:33:28 | -40 |

**Solution:**

(a) Plotting the points 5 marks, graph labelling etc. 2 marks, **best fit** curve 2 marks.




9 M

From the graph, orbital period of star A is $T_A$=120 min. 1 M

(b) Velocity semi-amplitude is $v_A = 393\,\mathrm{km\,s^{-1}}$, 1 M

Centripetal force on star A orbiting star B is equal to gravitational attraction between them

$$\therefore \frac{m_A v_A^2}{r} = \frac{G M_B m_A}{r^2}$$

$$\therefore M_B = \frac{v_A^2 \times r}{G}$$

$$v_A = \frac{2\pi r}{T_A}$$

$$\therefore r = \frac{v_A T_A}{2\pi}$$

$$M_B = \frac{v_A^3 T_A}{2\pi G}$$ 2 M

$$= \frac{(393 \times 10^3)^3 \times 120 \times 60}{2\pi \times 6.674 \times 10^{-11}}$$

$$\therefore M_B = 1.04 \times 10^{30}\,\mathrm{kg} \approx 0.52\ M_\odot$$ 2 M

If we allow a $\pm 3$ min error in period, and $\pm 10\,\mathrm{km\,s^{-1}}$ in velocity semi-amplitude, then the range of $M_B$ is $0.93 \times 10^{30}$ kg to $1.15 \times 10^{30}$ kg.

(c) The plot is not symmetric, because the whole system is moving towards us. From the best fit, $v_{CM} \approx 21\,\mathrm{km\,s^{-1}}$ 3 M

8. In this question, we will investigate construction of large mirrors of modern telescopes.

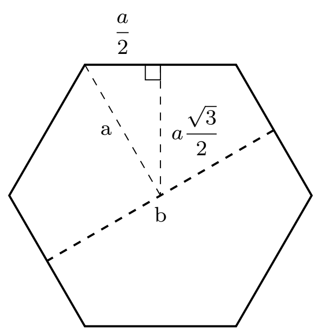
(a) (2 marks) Typical large modern telescopes do not have a single (monolithic) mirror. Instead, the mirror is divided in many small hexagonal concave mirror segments. Suppose one such telescope has segments with individual focal length of 30 m and length of their hexagonal side is 87.5 cm. What is the distance between two parallel edges of such a mirror segment?

(b) (6 marks) To avoid image distortion (due to spherical aberration), the telescope mirrors are made in the shape of paraboloid of revolution (a parabola rotated around its axis). Let us take one such mirror facing upwards and observing a star at zenith (exactly overhead). As you may be aware, such a parabola follows equation of $x^2 = 4ay$, where $a$ is the distance of the focus of the parabola from its vertex. You may also recall that slope of a tangent to the parabola at any point $(x_0, y_0)$ is given by $x_0/2a$.
Show that all rays from the star will converge at the focus of the parabola.

(c) (3 marks) In part (b) above, show that all these rays will arrive at the focus in same phase.

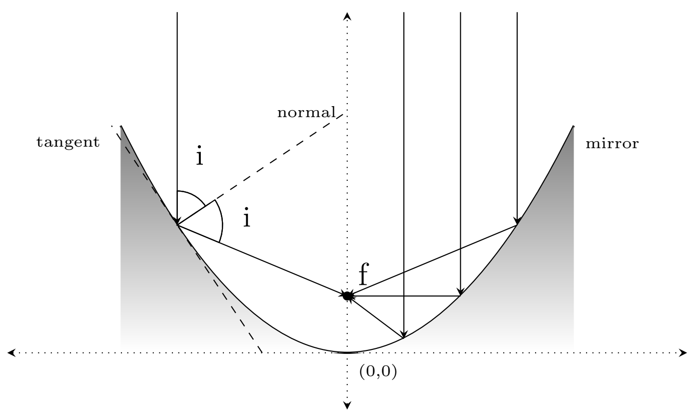
**Solution:**

(a) Side of each mirror segment = a = 0.875 m
Distance between opposite edges of segment ($b$) is given by,




$$b = 2 \times a \sin 60^\circ$$
$$b = 2 \times a\frac{\sqrt{3}}{2}$$
$$b = a\sqrt{3} = 0.875 \times \sqrt{3} = 1.516\,\text{m}$$ 2 M

$\therefore$ Distance between opposite edges of segment is 1.516 m.




(b) Product of slopes of two perpendicular lines is always $-1$. Therefore, for this parabolic mirror, as the slope at any point is $x_0/2a$, the slope of normal at that point will be $-2a/x_0$. 1 M

As the normal makes an angle '$i$' with the vertical and the reflected ray makes an angle '$2i$' with the vertical, 1 M

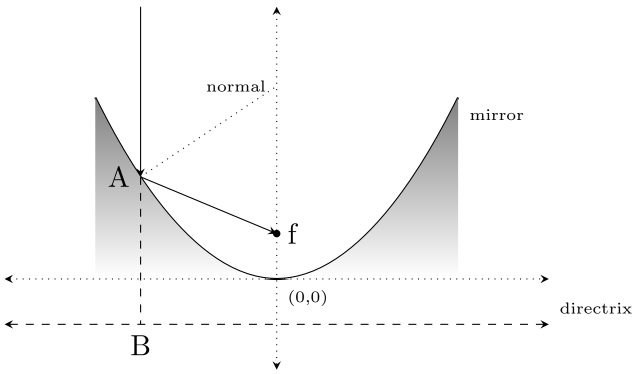
$$\tan(90 - i) = \frac{1}{\tan i} = \frac{-2a}{x_0}$$
$$\tan(2i) = \frac{2\tan i}{1 - \tan^2 i}$$
$$\tan(90 - 2i) = \frac{1}{\tan(2i)} = \frac{1 - \tan^2 i}{2\tan i}$$
$$= \frac{1 - \left(\frac{-x_0}{2a}\right)^2}{2\left(\frac{-x_0}{2a}\right)}$$
$$\tan(90 - 2i) = \frac{4a^2 - x_0^2}{4a(-x_0)}$$ 2 M

Now, we wish to find y-coordinate of the point where the reflected ray meets the principle axis. Note that the original point $(x_0, y_0)$ is also on the reflected ray.

$$(y_p - y_0) = m(x_p - x_0)$$
$$y_p = \tan(90 - 2i)(0 - x_0) + y_0$$
$$= \frac{4a^2 - x_0^2}{4a\cancel{(-x_0)}}\cancel{(-x_0)} + \frac{x_0^2}{4a}$$
$$= \frac{4a^2 - \cancel{x_0^2} + \cancel{x_0^2}}{4a} = \frac{4a^2}{4a}$$
$$y_p = a$$ 2 M

Thus, from any random point $(x_0, y_0)$ on the parabola, the reflected ray reaches, $(0, a)$, which is the focal point. Thus, all reflected rays are focussed at one point.

(c) See the figure below. As per the definition of parabola, $l(AB) = l(Af)$. 2 M
Thus, the rays meeting at focus, travel same distance as if they were travelling to the directrix. Thus, all rays falling on the mirror vertically travel same path length through the same medium and hence are in phase. 1 M



# Note for IAOSP candidates

- **Blackbody -**
  - emits as much energy as it absorbs,
  - emits in all directions equally.

  The thermal energy radiated by a blackbody per unit time per unit area is proportional to the fourth power of its absolute temperature and is given by

  $$\frac{L}{A} = \sigma T^4 \quad \mathrm{Jm^{-2}\ s^{-1}}$$

  where $L$ is luminosity, $A$ is area, $T$ is absolute temperature and $\sigma$ is Stefan's constant.