Indian National Astronomy Olympiad – 2010

Senior Category   Roll Number:
Question Paper

INAO – 2010   Date: 30th January 2010
Duration: **Three Hours**   Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.

- Before starting, please ensure that you have received a copy of the question paper containing total 3 pages (6 sides).

- In Section A, there are 10 multiple choice questions with 4 alternatives out of which only 1 is correct. You get 3 marks for each correct answer and **-1 mark for each wrong answer**.

- In Section B, there are 4 multiple choice questions with 4 alternatives each, out of which any number of alternatives may be correct. You get 5 marks for each correct answer. No marks are deducted for any wrong answers. **You will get credit for the question if and only if you mark all correct choices and no wrong choices.** There is no partial credit.

- For both these sections, you have to indicate the answers on the page 2 of the answer sheet by putting a × in the appropriate box against the relevant question number, like this:

  Q.NO. (a) (b) (c) (d)   Q.NO. (a) (b) (c) (d)
   22   □   ⊠   □   □   **OR**   35   ⊠   ⊠   □   □

  Marking a cross (×) means affirmative response (selecting the particular choice). Do not use ticks or any other signs to mark the correct answers.

- In Section C, there are 5 analytical questions totaling 50 marks.

- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.

- No computational aides like calculators, log tables, slide rule etc. are allowed.

- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.

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Useful Physical Constants

Mass of the Earth	$M_E \approx$	5.97×10^{24} kg
Radius of the Earth	$R_E \approx$	6.4×10^6 m
Mass of the Sun	$M_\odot \approx$	1.99×10^{30} kg
Radius of the Sun	$R_\odot \approx$	7×10^8 m
Radius of the Moon	$R_m \approx$	1.7×10^6 m
Speed of Light	$c \approx$	3×10^8 m/s
Astronomical Unit	1 A. U. $\approx$	1.5×10^{11} m
Gravitational Constant	$G \approx$	6.67×10^{-11} m³/(Kg s²)
Speed of Sound in Air	$c_s \approx$	330 m/s
Inclination of the Earth's Axis	$\epsilon \approx$	23.5°

Space for Rough Work

Section 1:Multiple Choice Questions
Part A: (10 Q × 3 marks each)

1. As seen from the Earth, the stars appear to twinkle, while the planets do not because,

    A. Light coming from the stars gets absorbed by interstellar dust.

    B. the stars are self luminous while the planets merely reflect the light.

    C. Angular sizes of the stars are much smaller than the planets.

    D. All the above.

A wooden cube of length 5 units is painted on all faces and then cut in 85 smaller cubes of varying sizes. The sides of each of the smaller cubes are some integer number of units. All bigger cubes are carved out of the corners of the original cube. Answer the following three questions:

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2. How many cubes have no side painted?

    A. 15   **B. 17**   C. 20   D. 27

Solution: If you cut the bigger cube in smaller cubes with all side of 1 unit, there will be 125 total cubes.

Each cube of size 2 will fuse 8 such smaller cubes together. i.e. the total number of cubes will reduce by 7.

Similarly, each cube of size 3 and 4 will reduce total number of cubes by 26 and 63 respectively.

Now, we have 85 total cubes, which is 40 less than 125. This means there is 1 cube of size 3 and 2 cubes of size 2 at three of the corners. Rest all are size 1.

If there were 125 smaller cubes of size 1, one can say the volume which is not touched by paint on any face is equivalent to a cube of size 3 i.e. 27 smaller cubes. However, the the bigger cubes carved out of corners include some of these inner cubes as well. 8 such inner smaller cubes are included in the size 3 cube and 1 each is included in the 2 size 2 cubes.

Thus, number of unpainted cubes $= 27 - 8 - 1 - 1 = 17$

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3. How many cubes have exactly 2 sides painted?

    A. 24   B. 27   C. 30   D. 36

Solution: To have exactly 2 sides painted, the smaller cube should be along any of the edges but not at the corner. There are total 12 edges to the original cube. For each edge, there will be 3 cubes satisfying the condition if all cubes were of size 1. How ever, there is a size 3 cube, which includes 2 such smaller cubes each

along 3 of the edges.
Similarly, the 2 size 2 cubes will each include 1 such smaller cube each along 3 of the edges.
Thus, total number of cubes $= 12 \times 3 - 2 \times 3 - 1 \times 3 - 1 \times 3 = 24$

4. If one was to cut the original cube (of size 5 units each side) with a condition that <u>all</u> corners would be occupied by the cubes bigger than 1 unit size, what will be the smallest total of cube pieces possible?

   A. 69   B. 64   C. 62   **D. 50**

   Solution: There are two possibilities which will satisfy this condition. Either all corners will have cubes of size 2 or else there will be 1 cube of size 3 and rest all corners will have cubes of size 2.
   There are total 8 corners.
   In first case total number of cubes will be $= 125 - 8 \times 7 = 69$
   In second case, total number of cubes will be $= 125 - 7 \times 7 - 26 = 50$

5. It is easier to balance on a faster bicycle moving along a straight line than a slower one because of

   A. conservation of linear momentum

   B. conservation of angular momentum

   C. conservation of linear momentum and angular momentum

   D. conservation of energy

6. On a nice sunny day in Chennai, Akshay saw a supersonic fighter plane flying parallel to the ground in the sky. As a student of Aeronautical Engineering, he knew off-hand the speed of the plane to be 1.25 Mach (i.e. 1.25 times the speed of sound in air). He could hear its sonic boom, 12 seconds after the plane flew directly overhead. What is the altitude of the plane?

   A. 3.7 Km   B. 4.0 Km   C. 5.3 Km   **D. 6.6 Km**

   Solution: Half angle of the cone of the sonic boom is given by $\sin \theta = \frac{v_s}{v_p}$, where v_p is velocity of the plane and v_s is the speed of the sound in air.

$$\sin\theta = \frac{v_s}{1.25 v_s} = 0.8$$
$$\therefore \cos\theta = 0.6$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{h}{v_p t}$$
$$\frac{0.8}{0.6} = \frac{h}{1.25 \times 330 \times 12}$$
$$h = \frac{4 \times 15 \times 330}{3}$$
$$= 6600 \text{ mt.} = 6.6 \text{ km}$$

7. If you hold a magnifying glass of focal length 10 cm in the sunlight and place a piece of paper at its focus, you can burn a hole in the paper. What could be the size of this hole?

   A. 10 mm   B. 5 mm   **C. 0.5 mm**   D. 0.1 mm

   Solution: If the magnification is given by M, Object distance, Object size, Image Distance, Image size by $u, O, v \& I$ respectively,

   $$M = \frac{v}{u} = \frac{I}{O}$$
   $$\frac{I}{7 \times 10^8} \approx \frac{f}{500 c}$$
   $$I \approx \frac{0.1 \times 7 \times 10^8}{500 \times 3 \times 10^8}$$
   $$\approx \frac{7}{15} \times 10^{-3} \text{ mt.}$$
   $$\approx 0.5 \text{ mm}$$

8. During an earthquake, an earthquake monitoring Centre observed that transverse waves traveling with speed 4.5 km/s arrived at the centre 3 minutes after the longitudinal waves traveling at 8.2 km/s. Deduce the approximate distance to the epicenter.

   A. 60 km   B. 220 km   C. 660 km   **D. 1800 km**

Solution:

$$
\begin{aligned}
8.2t &= 4.5 \times (t + 180) \\
3.7t &= 810 \\
D &= 8.2t = 8.2 \times \frac{810}{3.7} \\
&= 2.22 \times 810 \\
D &\approx 1800 \text{km}
\end{aligned}
$$

9. A certain person nicknamed "Enthu", encountered an automatic staircase (i.e. escalator) at a shopping complex, which was moving upward at a constant rate. Just for the fun of it he decided to walk up this escalator at the rate of one step a second. Twenty steps brings him to the top. Next day he goes up at two steps a second and reaching the top in 32 steps. How many steps are there in the escalator?

   A. 40   B. 60   C. 64   **D. 80**

   Solution: Let the escalator rise with the speed of 'n' steps per second.
   On first day, Enthu reaches the top after 20 manual steps i.e. in 20 seconds.
   On second day, Enthu reaches the top after $32 / 2 = 16$ seconds.
   Thus, total number of steps,

$$
\begin{aligned}
20 \times (n + 1) &= 16 \times (n + 2) \\
4n &= 32 - 20 = 12 \\
\therefore n &= 3 \\
\therefore \text{Steps} &= 20 \times (3 + 1) \\
&= 80
\end{aligned}
$$

10. Two stars of masses M and $3M$ respectively are going around each other, in near circular orbits, with period T. The separation between them is given by $D = \sqrt[3]{\frac{kGMT^2}{\pi^2}}$. The value of k is,

    A. 0.5   **B. 1**   C. 1.5   D. 3

    Solution: The Centre of mass of the system will be located at distance $\frac{D}{4}$ from

the heavier mass. Centripetal force is provided by the mutual gravitation.

$$\frac{G\,M\,(3M)}{D^2} = (3M)\frac{D}{4}\omega^2$$

$$\frac{G\,M}{D^2} = \frac{D}{4}\frac{4\pi^2}{T^2}$$

$$D^3 = \frac{GMT^2}{\pi^2}$$

$$D = \sqrt[3]{\frac{GMT^2}{\pi^2}}$$

Section B: (4 questions × 5 marks each)

11. If we throw a ball in a shallow water tank, propagation velocity of ripples on surface of the water will depend on

    A. surface tension of the water

    B. depth of the water tank

    C. density of the water

    D. height from which the ball was dropped

    Solution: Note: The propagation velocity will vary as per the depth of the water, provided the water is not too deep. However, the derivation for this effect includes concepts beyond the level of participants. Thus, the accepted solution for this question would be 'a' and 'c' must be selected, 'd' must not be selected and 'b' would be acceptable both ways.

12. Amit decided to experiment with cannon ball by making it hollow and filling water inside it. He then punched few holes in it. After the ball was fired horizontally, he was expecting to see water jets coming out from some of the holes. Which of the following locations of holes will allow water jet to come out?

    A. Front, bottom and sides   B. Back, top and sides   C. back and bottom
    D. None of the holes

    Solution: Horizontally, all particles (i.e. cannonball and water particles) are flying with the same velocity. Hence water cannot come out of front / back /side holes. Vertically, all particles are experiencing free fall. Thus, gravity cannot exert any additional pressure on water and neither can water be slower than the cannonball. Hence no water can come out from top or bottom.

13. In the progress of Astronomy over the ages, we find several instances of startling new observations changing our ideas about the Universe and lead to new theories. Listed below are some milestones in the history of Astronomy and observations which necessitated them. Pick the <u>correct</u> statement(s).

A. **Ptolemy was aware of retrograde motion of planets when he gave his model of the solar system.**

B. **Newton was aware of Kepler's Laws when he gave his Law of Gravitation.**

C. Einstein felt need to modify Newton's theory of Gravitation to explain the expansion of the Universe.

D. Existence of Cosmic Microwave Background Radiation (CMBR) led to the creation of the Big Bang Theory.

Solution: Ptolemy needed epicyclic model of the solar system as that was the only way to fit observed motions of the planets including the retrograde motion. Kepler's laws enabled Newton to deduce $\frac{1}{r^2}$ nature of the law of gravitation.

Einstein gave General theory of relativity in 1916 and assumed the Universe to be non-expanding as per then norm. Hubble discovered the expansion of the Universe in late 20's forcing Einstein to revisit his own theory.

Existence of CMBR was one of the key predictions of the big bang theory. The theory was formulated before CMBR was discovered.

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14. Two simple pendula are hung close to each other on a thin, rigid support and are allowed to oscillate independently in planes parallel to each other. They have periods 3 seconds and 7 seconds respectively with same amplitudes and are initially released from the opposite extreme positions of each other. At which of the following times the threads of the two pendula will be coplanar again?

A. **1.05 seconds**   B. **7.88 seconds**   C. 10.50 seconds   D. 23.10 seconds

Section C: Analytical Questions

α. An alien civilisation on a star far far away came to know about the Astronomy Olympiad Examination and wanted to test smartness of the students. They sent following two coded secret messages. Decode them.

(a) (5 marks) First is a pictorial message in black and white colour. They sent it on radio waves in the form of ones and zeros. Find out what the picture says.
00000 00000 00000 00000 00000 00000 11111 01100 00100 00100 00011 10000 01000 10100 01000 10100 01101 10000 10001 00100 10001 11000 10001 00001 00010 00101 00100 01001 10110 01111 10100 00110 10000 01001 11000 00000 00000 00000 00000 00000 000

(b) (5 marks) Surprisingly, the aliens are also proficient in English and the following coded message is actually a sentence in English language. In the answer sheet, write down the coded sentence and also the true meaning of each alphabet in the code.

"Up tpmwf uif qsfwjpvt tvcrvftujpo uijol pg uif nfttbhf tfou cz uif bsfdjcp ufmftdpqf up bmjfot."

Solution: In 1974, a pictographic message was sent to outer space by radio waves using Arecibo telescope. The total number 'digit's in the message was product of two prime numbers. It was argued that any intelligent being can rearrange the grid in a rectangle with sides measuring these two prime numbers and then use blank square for '0' and filled square for '1' to reveal the picture.

In the present message same idea is used. Total number of digits is 203 which is a semiprime number; i.e. it can only be divided as 29×7. If the grid is rearranged in this way, following pattern is seen:
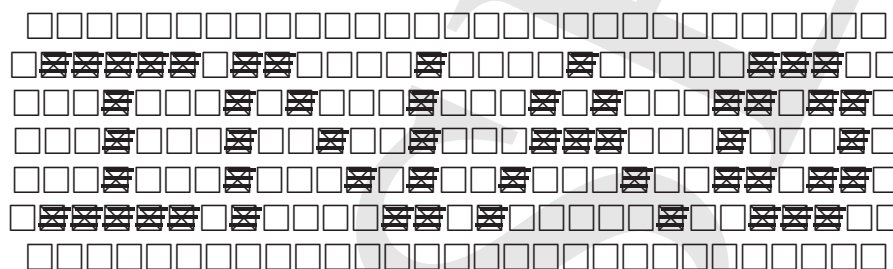
Marking Scheme:

- Realising total number of digits are 203, which is a semi-prime number. **(2 marks)**

- Attempt to draw either 29×7 or 7×29 grid. **(1 mark)**

- picking the message "INAO". **(2 marks)**

Part (b) is a substitution cipher.
The word "uif" appears thrice in the sentence. Obviously, it stands for "the". Looking at the two words, the substitution rule is inferred as "every letter is substituted by the next letter" to create the coded message. Deciphering,

"To solve the previous subquestion think of the message sent by the arecibo telescope to aliens." Marking Scheme:

- Mapping the alphabets and deciphered Sentence. **(5 marks)**

- Mapping the alphabets and Deciphered Sentence, but with small syntactic errors. **(4.5 marks)**

- Mapping the alphabets only. **(4 marks)**

- Deciphered Sentence but Alphabet mapping is not given. **(3 marks)**

- Only partial decipherment with more than 10 letters. **(2 marks)**

- Only partial decipherment with 7-10 letters. **(1.5 marks)**

- Only partial decipherment with "the" deciphered correctly. **(1 mark)**

β. (8 marks) In the following table, the first column gives various optical phenomena / instruments and the top row gives various optical effects which may help in explaining them. In the answers sheet, tick the correct effect(s) involved in each phenomena in appropriate rows.

	Appreciable Dispersion	Internal Reflection	Reflection	Refraction	Scattering
Blue Sky					
Mirage					
Rainbow					
Smooth Convex Mirror					
Thick Concave Lens					

Solution:

	Appri. Disp.	Int. Refle.	Refle.	Refra.	Scat.
Blue Sky					✓
Mirage		✓		✓	
Rainbow	✓	✓		✓	
Sm. Conv. Mirror			✓		
Th. Conc. Lens	✓			✓	

- Blue Sky (**1 mark**): One mark for scattering. -0.5 for each additional tick mark.

- Mirage (**2 marks**): One each for internal reflection and refraction. 0 marks if internal reflection is not ticked.

- Rainbow (**2 marks**): One for internal reflection; 0.5 each for dispersion and refraction. 0 marks if internal reflection is not ticked.

- Mirror (**1 mark**): One mark for reflection. 0 if any additional / other tick marks.

- Thick Lens (**2 marks**): One each for dispersion and refraction.

Any options not mentioned above, do not carry either positive or negative credit.

γ. Study the following image of the night sky (bigger version is printed in your answer-sheet) for <u>mid-night</u> on a certain day at a certain place and answer the questions below it. All answers must be marked / written on the answer-sheet.
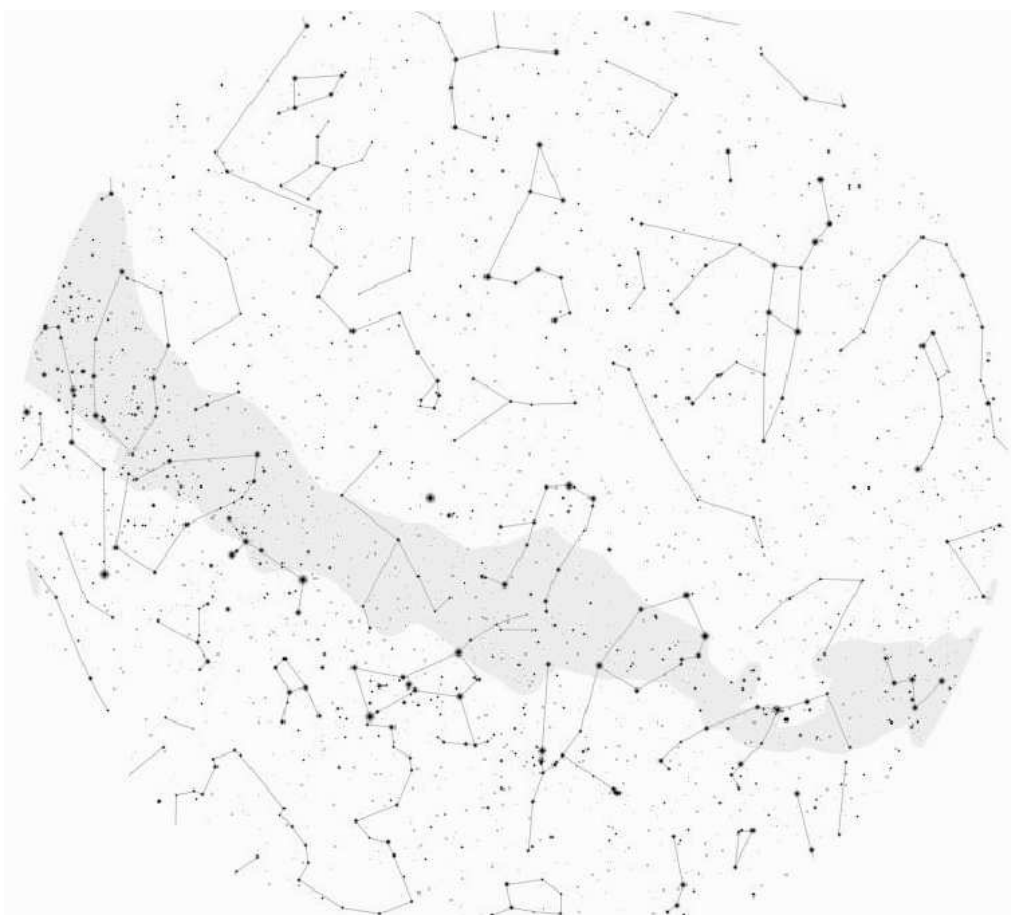
(a) (2 marks) Mark all the four directions on the map.

(b) (1 mark) Is this place in northern part of India or the southern part?

(c) (3 marks) Mark and name 2 constellations each from the following two lists:

- **0.5 marks each; any 2:** Orion, Ursa Major, Taurus, Leo, Cygnus, Scorpio
- **1 mark each; any 2:** Hydra, Corvus, Aquarius, Cancer, Canis Major, Aries

(d) (2 marks) Sketch rough band of ecliptic i.e. apparent path of the Sun, the Moon and all the planets in the sky.

(e) (2 marks) In which month the sky will appear like this at this time? Give reason in one line.

Solution: (a) From Top clockwise: East, North, West, South. Note the position of Ursa Minor Constellation (i.e. Pole star) to the right of the map. Further, Taurus is at the bottom and Leo is at the top. **(0.5 marks per direction)** If the direction pointers are off by small angle (less than 10°), overall -0.5 marks.
(b) Ursa Minor is very close to Horizon. Hence the latitude of the place is not very high. Thus, the place is in Southern India. **(-0.5 if wrong reason is mentioned.)**
(c) Cygnus, Scorpio, Aquarius are not on the map. Rest are clearly visible.

(d) The band of ecliptic should pass through all the zodiac signs (i.e. Aries to Virgo). Better precision than this is not expected. For every zodiac sign significantly away from the ecliptic, -0.5 marks (minimum zero).
(e) At mid-night, Cancer is nearly at Zenith. Hence the Sun is roughly 6 zodiac signs away i.e. in Capricorn. Thus, it will reach Vernal Equinox (in Pisces) in roughly two months. Hence, current date is about two months before 21st March. Thus, the current month is January. 0.5 marks for getting month (December - February accepted). Reason 1.5 marks.

δ. Mayank visited a place located at latitude ϕ and longitude $82.5°E$ on 21st June. He observed that at local noon, shadow of a one meter stick standing vertically on the ground was 26.8 cm long due south.

(a) (5 marks) Find latitude of the place.

(b) (5 marks) Find the day on which the shadow of this stick at the local noon will be longest and find length and direction of that shadow.

Note: $\sin^{-1}(0.268) = 15.5°$, $\cos^{-1}(0.268) = 74.5°$, $\tan^{-1}(0.268) = 15.0°$, $\tan(2°) \approx \frac{1}{30}$ and $\sqrt{3} = 1.73$

Solution: Shadow is 0.268 m due South on the day of the Summer solstice. This means the Sun is to the North of Zenith, i.e. the place is south of tropic of cancer i.e. in southren India. **(1 mark)**
On that day, the Sun is directly above the tropic of cancer, i.e. $90° - 23.5° = 66.5°$ away from the North Celestial Pole. **(1 mark)**
If the altitude of the Sun at the local noon is θ,

$$
\begin{aligned}
\tan\theta_{\text{Jun}} &= \frac{1}{0.268} = \frac{1}{\tan(15°)} \\
&= \tan(75°) \\
\therefore \phi &= 75° - 66.5° \\
\therefore \phi &= 8.5° \text{(3 marks)}
\end{aligned}
$$

In part (b) For a northern hemisphere place, the longest shadow of the stick will be cast on the Winter Solstice day. The Sun at local noon will be 47° further south than its position on the Summer Solstice day. Hence the Sun will be $180° - 75° - 47° = 58°$ high on the South of the Zenith. **(2 marks)**

$$
\begin{aligned}
l &= \frac{1}{\tan 58^\circ} \\
&= \tan 32^\circ \\
&= \tan(30^\circ + 2^\circ) \\
&= \frac{\tan 30^\circ + \tan 2^\circ}{1 - \tan 30^\circ \tan 2^\circ} \\
&\approx \frac{\frac{1}{\sqrt{3}} + \frac{1}{30}}{1 - \frac{1}{\sqrt{3}} \times \frac{1}{30}} \\
&\approx \frac{30 + \sqrt{3}}{30\sqrt{3} - 1} \\
&\approx \frac{31.73}{51.9 - 1} \approx \frac{31.73}{51} \\
&\approx 0.62\text{m}
\end{aligned}
$$

The Shadow will be 0.62 m long due North. **(3 marks)**

ϵ. Sketch approximate graphs for the following situations:

(a) (4 marks) See the figure below. A tank of water (height of water column b) is kept on a electronic weighing scale. A metal cube (side a and density ρ) is hung from a spring balance and the spring balance is slowly lowered into the tank till the cube reaches the bottom of the tank. The distance of separation between the bottom of the tank and bottom of the cube is denoted by h with initial value h_0.
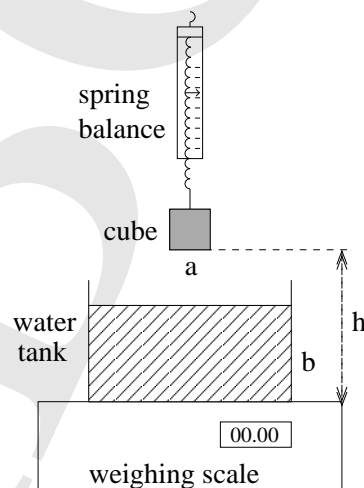

Sketch the graph of reading on the spring balance as a function of h.

(b) (3 marks) For the situation above, sketch the graph of reading on the electronic scale as a function of h.

(c) (2 marks) For the situation above, sketch the graph of sum of the reading on the electronic scale and the reading on the spring balance as a function of h.

(d) (3 marks) For a typical primary mirror used in Newtonian telescope, sketch a

graph of object distance, u versus image distance, v. All physically measurable lengths should be taken as positive.

In all cases, mark the significant points on the graph and give their coordinates.

Solution:
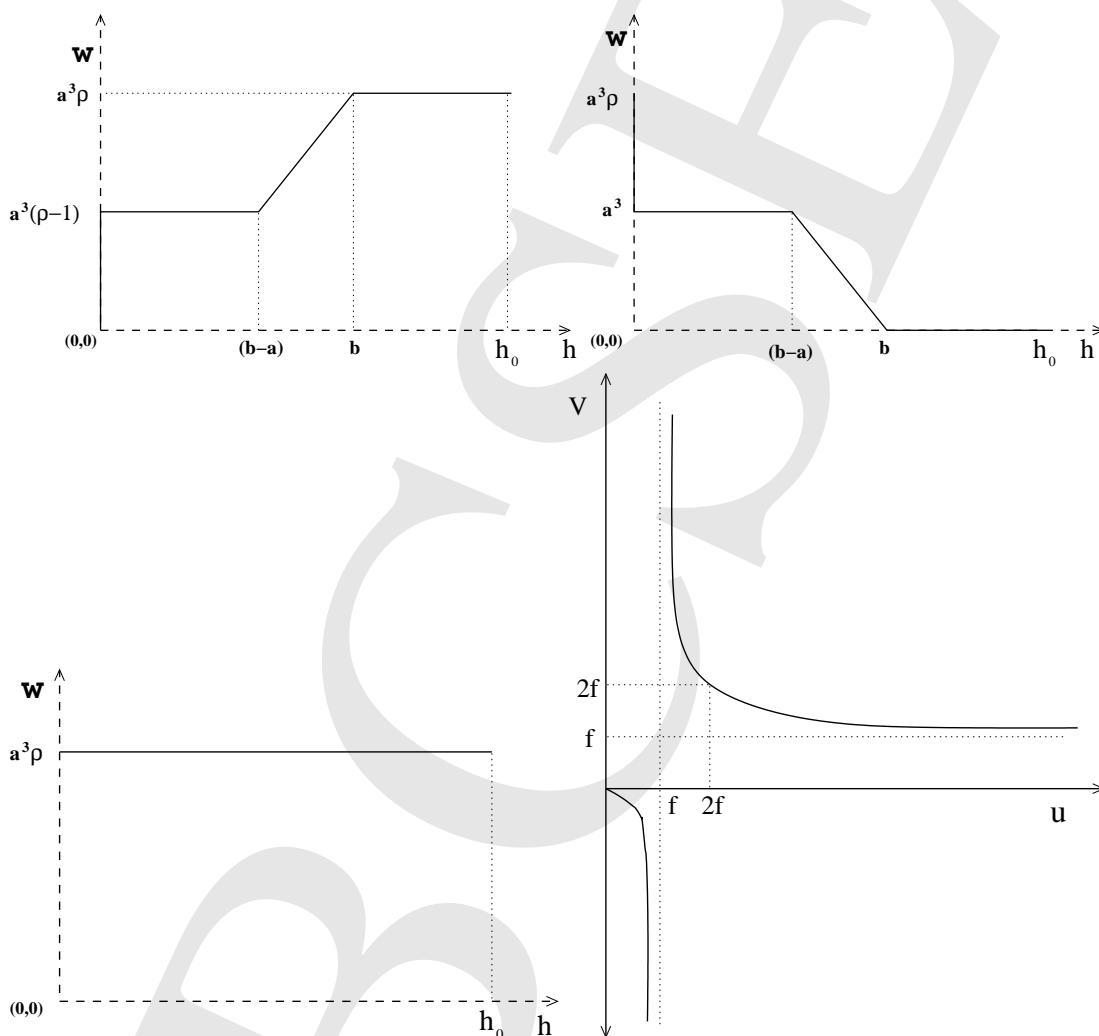
- Partial marks given on basis of shape of the curve, neatness of figure, proper nomencleture (axis titles etc.), proper markingof points and writing their co-ordinates etc.

- In (a), (b) and (c), it is acceptable to take mass of water as m instead of zero. No points cut.

- The y-coordinates can be either in mass units or in force units (multiplication by g). No points cut.

- Density of water can be written as ρ_w or its CGS values (i.e. 1) or its MKS value (i.e. 1000). No points cut.

- In (d), the curve between $u = 0$ and $u = f$ may be plotted in both quadrant 1 and quadrant 2. No points cut.

Space for Rough Work