Indian National Astronomy Olympiad – 2012

Question Paper   Roll Number: ☐

INAO – 2012   Date: 28th January 2012
Duration: **Three Hours**   Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 4 pages (8 sides).
- In Section A, there are 10 multiple choice questions with 4 alternatives out of which only 1 is correct. You get 3 marks for each correct answer and **-1 mark for each wrong answer**.
- In Section B, there are 2 multiple choice questions with 4 alternatives each, out of which any number of alternatives may be correct. You get 5 marks for each correct answer. No marks are deducted for any wrong answers. **You will get credit for the question if and only if you mark all correct choices and no wrong choices.** There is no partial credit.
- For both these sections, you have to indicate the answers on the page 2 of the answer sheet by putting a × in the appropriate box against the relevant question number, like this:

Q.NO.	(a)	(b)	(c)	(d)		Q.NO.	(a)	(b)	(c)	(d)
22	☐	☒	☐	☐	**OR**	35	☒	☒	☐	☐

  Marking a cross (×) means affirmative response (selecting the particular choice). Do not use ticks or any other signs to mark the correct answers.
- In Section C, there are 5 analytical questions totaling 60 marks.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- No computational aides like calculators, log tables, slide rule etc. are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
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V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Useful Physical Constants

Mass of the Earth	M_E	$\approx$	6×10^{24} kg
Radius of the Earth	R_E	$\approx$	6.4×10^{6} m
Mass of the Sun	$M_\odot$	$\approx$	2×10^{30} kg
Radius of the Sun	$R_\odot$	$\approx$	7×10^{8} m
Radius of the Moon	R_m	$\approx$	1.7×10^{6} m
Distance to the Moon	d_m	$\approx$	3.84×10^{8} m
Speed of Light	c	$\approx$	3×10^{8} m/s
Astronomical Unit	1 A. U.	$\approx$	1.5×10^{11} m
Gravitational Constant	G	$\approx$	6.67×10^{-11} m^3/(Kg s^2)
Inclination of the Earth's Axis	ϵ	$\approx$	23.5°
Gravitational acceleration	g	$\approx$	10 m/s^2

Space for Rough Work

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Section 1:Multiple Choice Questions

Part A: (10 Q × 3 marks each)

1. If the square of your age in seconds gives the age of the Universe in seconds, then

   A. you haven't started to walk yet

   B. you are in primary school

   C. you are a young adult

   D. you were born before British left India

Solution: Age of the Universe is about 14 billion years. If your age in years is Y, then,

$$\begin{aligned} (Y \times 365.25 \times 86400)^2 &= 14 \times 10^9 \times 365.25 \times 86400 \\ \therefore Y^2 &= \frac{14 \times 10^9}{365.25 \times 86400} \\ &\approx \frac{14}{28} \times 10^3 \approx 500 \\ \therefore Y &\approx \sqrt{500} \approx 22 \text{ years} \end{aligned}$$

Thus, your age is in early 20s.

2. If $f(x+y) = f(x)f(y)$ and $f(2) = 5$, then the value of $f(-2)$ is

   A. 5 B. 1 C. 0.25 **D. 0.2**

Solution:

Solution 1:
$f(4) = f(2)f(2) = 25$ and $f(2) = f(4-2) = f(4)f(-2) = 25f(-2)$
Thus: $f(-2) = \frac{f(2)}{25} = \frac{5}{25} = 0.2$

Solution 2:
$f(2) = f(2+0) = f(2)f(0)$ implying $f(0) = 1$
Thus, $f(0) = f(2-2) = f(2)f(-2)$
Giving, $f(-2) = \frac{f(0)}{f(2)} = \frac{1}{5} = 0.2$

3. A container with uniform rectangular cross-section and weight M kg, falls from a cargo ship into the sea and is floating with x part of its height under water (x is a fraction less than 1). Two persons from a nearby boat, each weighing m kg board on

the container resulting it to go down such that the top surface of the container levels exactly with the water level. Find the ratio $\frac{M}{m}$ in terms of x.

A. $\frac{(1-x)}{2x}$ B. $\frac{x}{(1-x)}$ C. $\frac{(1-x)}{x}$ **D.** $\frac{2x}{(1-x)}$

Solution: If V is volume of the container,

$$\begin{aligned} M &= xV \\ \& \; M+2m &= V \\ \therefore M+2m &= \frac{M}{x} \\ 2mx &= M(1-x) \\ \therefore \frac{M}{m} &= \frac{2x}{(1-x)} \end{aligned}$$

4. An electron moving uniformly in space is neither deflected nor accelerated over a long distance. Which of the following statements may describe the local conditions?
   A. Both electric and magnetic fields are necessarily zero simultaneously.
   B. The electric field is necessarily zero.
   C. The magnetic field is necessarily zero.
   D. Neither of them are necessarily zero.

Solution: The Lorentz force equation is given by,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

We require $\vec{F}$ to be zero. The electric and magnetic fields can be so adjusted that their net effect cancels out, and the electron continues its path.

5. Pradip once observed image of the Sun on the platform of CST station (Mumbai's main railway station). He realised that the sunrays forming the image are entering through a tiny triangular shaped window in the high ceiling. He measured the diameter of the image to be 0.175 m. Find the height of the ceiling.

A. 12.5 m **B. 18.75 m** C. 22.5 m D. 37.5 m

Solution: h = height of the ceiling; $D_{se} = 1.5 \times 10^{11}$ m ;w = diameter of image; $d = 14 \times 10^8$ = diameter of sun.

$$\begin{aligned} \frac{h}{w} &\approx \frac{D_{se}}{d} \\ h &\approx .175 \times \frac{1.5 \times 10^{11}}{14 \times 10^8} \\ h &\approx 18.75m \end{aligned}$$

6. The most energy efficient direction of projection of rockets from the earth surface is

   A. Eastwards B. Westwards C. Northwards D. Vertically upwards

> **Solution:** The earth rotates from west to east. If the rocket is projected in a eastward direction, the earth's motion acts as a boost for the projection.

7. A car travels the first 20 km eastwards with 30 km/hr, next 20 km northwards with 40 km/hr and then 20 km westwards with 50 km/hr. The average velocity of the car for the journey is

   A. 10 km/hr B. $\frac{10}{9}$ km/hr **C. $\frac{600}{47}$ km/hr** D. $\frac{400}{47}$ km/hr

> **Solution:** Total displacement is 20 km only.
>
> $$\begin{aligned} \text{Avg.Velocity} &= \frac{\text{total displacement}}{\text{total time}} \\ \text{totaltime} &= \frac{20}{30} + \frac{20}{40} + \frac{20}{50} \\ &= \frac{47}{30}\text{hr} \\ \text{Avg.Velocity} &= \frac{20}{\frac{47}{30}} \\ &= \frac{600}{47} \end{aligned}$$

8. The Sun is found to be setting exactly at 6.00 pm on a given day. If the Earth's atmosphere was only half dense as it is then the sunset would have occurred

   A. Slightly later than 6.00 pm

   B. Slightly earlier than 6.00 pm

   C. Exactly at 6.00 pm

   D. It depends on the latitude of the place.

> **Solution:** We "see" the sunset a few minutes after geometric sunset as the solar disk remains visible even after going below horizon due to refraction. If the atmosphere is rarer, the sunrays will get refracted less. Thus, this additional period of visibility of solar disk is reduced. Hence, we will "see" sunset slightly earlier.

9. A knock-out tennis tournament begins with a total of 193 players. In each round, if the number of players is odd, then one player gets direct bye to the next round. No

player can get byes in two consecutive rounds. How many matches must be played in the tournament before the winner is decided?

A. 386 **B. 192** C. 169 D. 97

Solution: The only way a player can go out of the tournament is by losing. Winner is a single player; thus, $193 - 1 = 192$ players must lose. Implying 192 matches.

10. **Four** metal rods all of identical dimensions and made of same material are welded together at a single point. The configuration is such that any two rods are oriented at 120°. The far ends of three of the rods are maintained at 90° temperature and that of the fourth rod is maintained at 30° temperature. The temperature of the junction point is

A. 45° B. 60° **C.** 75° D. None of these.

Solution: For equilibrium condition, heat flow in and out of junction should be same. Let k represent some constant representing heat content, which may depend on dimensions and material of the rods.

$$\begin{aligned} k(T-30) &= 3k(90-T) \\ 4T &= 300 \\ T &= 75 \end{aligned}$$

Section B: (2 questions × 5 marks each)

11. Consider the following statements:

- A central eclipse is the one where central point of lunar disk exactly passes over the central point of solar disk.
- An eclipse is called a total Eclipse if it is seen as total from at least some point on the earth.
- An eclipse is called a partial Eclipse if it is **not** seen as total / annular from any point on the earth.

Now choose the **incorrect** statement/s from below:

A. All central eclipses are total.

B. All total eclipses are central.

C. All partial eclipses are non-central

D. All non-central eclipses are partial

Solution: First one is incorrect as some central eclipses can be annular only.
Second one is incorrect as one can get a total eclipse even when the two disks are slightly off-center w.r.t. to each other.
Third one is correct.
Fourth one is incorrect for the reason explained above regarding the second option.

12. Consider a pendulum with inextensible string with a lightweight magnetic bob. Underneath this arrangement iron dust is spread out. Now the bob of the pendulum starts swinging. Which of the following might be a susequent observation(s)?

    A. The maximum height reached by the bob will start increasing.

    B. The tension in the string will start increasing.

    C. The period of oscillation of the bob will start decreasing.

    D. The angular momentum of the bob remains constant.

Solution: As the oscillation starts the bob will slowly pick up iron dust and its mass will increase. As the velocity of the bob at the hightest point is zero, by conservation of energy, height of the highest point must decrease.
As the mass increases, the tension in the string will increase too.
Since the string is inextensible the time period will remain the same.
But angular momentum, $L = I\omega$ will change, as I changes.(I is moment of inertia).

Section C: Analytical Questions

α. (10 marks) In the night sky, the constellations depict several shapes of animate objects (species like birds, animals, etc) or man made instruments. List 20 such figures / instruments. The list should contain at least 4 instruments. Write both the object and the corresponding constellation. Exact name of the constellation is not necessary. The constellations must be from International Astronomical Union's standard list of 88 constellations (i.e. Indian / Chinese / Mayan constellations not accepted).

Solution:

Animals (easy): bull (Taurus), ram (Aries), eagle (Aquila), bear (Ursa Major / Ursa Minor), lion (Leo), crab (Cancer), scorpion (Scorpio), dog (Canis Major / Canis Minor), fishes (Pisces), swan (Cygnus), dolphin (Delphinus), snake (Serpens), crow (Corvus), man (Orion / Bootes etc.)

Animals (difficult) - : fly (Musca), hare (Lepus), Wolf (Lupus), fox (Vulpecula), peacock (Pavo), dove (Columba), Emu (camelopardalis), small horse (Equuleus), swordfish (Dorado), crane (Grus), lizard (Lacerta), Chamaeleon, hornbill (Tucana)

Animals (mythical): water goat (Capricornus), centaur (Centaurus), dragon (Draco), winged horse (Pegasus), a mythical snake (Hydra), a sea monster (Cetus), a bird (Apus), Phoenix

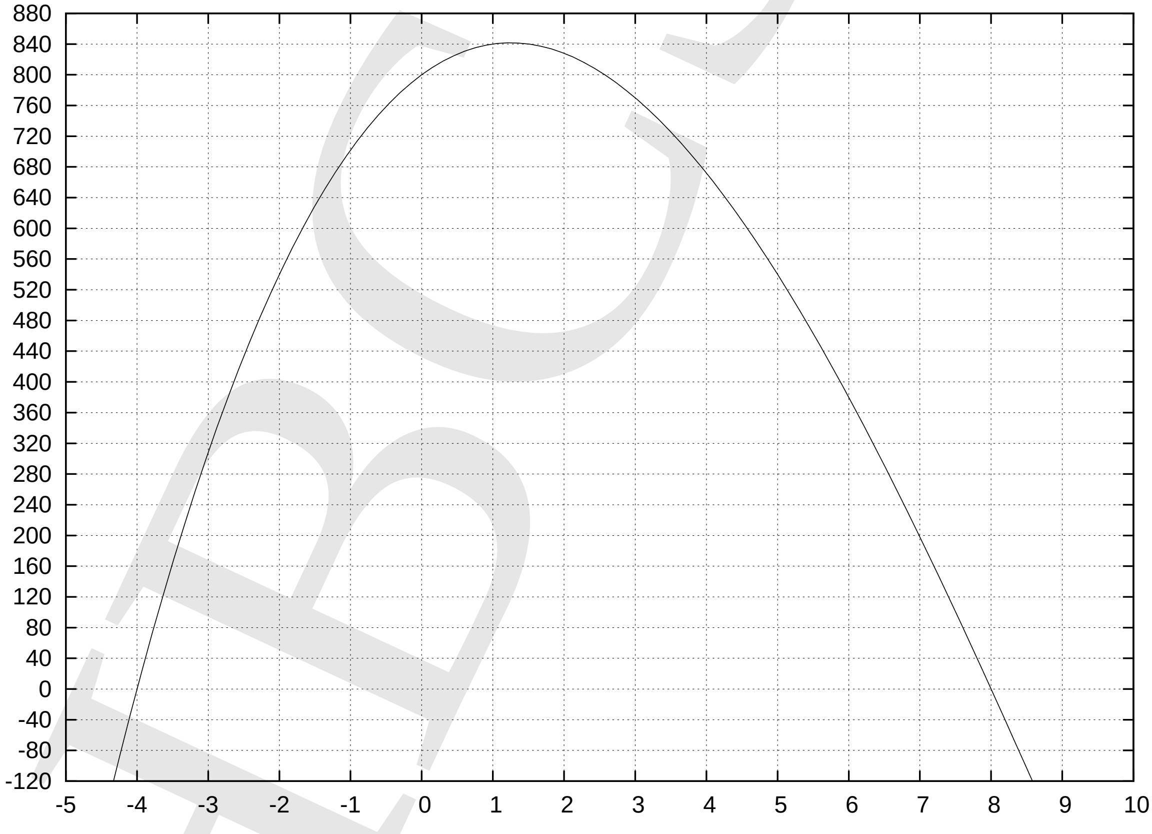
instruments: Telescopium, scales (Libra), Microscopium, Sextans, directional compass (Pyxis), Table (Mensa), cross (Crux), Crown (Corona Australis / Corona Borealis), harp (Lyra), arrow (Sagitta), shield (Scutum), easel (Pictor), air pump (Antila), compasses (Circinus)

Each correct creature / instrument: 0.5 marks. Maximum 16 creatures allowed.

β. From the graph below, find:

(a) (7 marks) The cubic polynomial describing the curve. [Note: $f(7) \neq 200$.]

(b) (5 marks) If this curve describes orbit of a comet with the Sun at point (1,0) and the comet crosses the x-axis exactly two months apart, find the approximate position of the comet 1 month since first crossing of the x-axis.



Solution: (a) From the graph, we note that $f(-4) = 0$, $f(8) = 0$, $f(0) = 800$

and $f(1) = 840$. Using these to find the cubic polynomial, **(2.5 marks)**

$$\begin{aligned}
f(x) &= ax^3 + bx^2 + cx + d \\
0 &= a(-4)^3 + b(-4)^2 + c(-4) + d = -64a + 16b - 4c + d \\
0 &= 8^3a + 8^2b + 8c + d = 512a + 64b + 8c + d \\
800 &= d \\
840 &= a + b + c + d \\
\therefore 40 &= a + b + c \\
\& \; 0 &= 576a + 48b + 12c \\
\therefore -c &= 48a + 4b = 4(12a + b) \\
\& \; 0 &= 64a + 8b + c + 100 \\
\therefore 0 &= 24b - 3c + 900 \\
\therefore c &= 8b + 300
\end{aligned}$$

Solving, we get,

$$f(x) = x^3 - 29x^2 + 68x + 800 \tag{1}$$

(b) We have to divide the area enclosed by the curve and the x-axis in 2 equal halfs as divided by the line from point (1,0), to get the position for the comet after one month.

Non-calculus solution: A rough counting of squares shows that the correct position would be approximately between $x = 2$ and $x = 2.5$.

Calculus based solution: If the x-coordinate of the comet after one month was x_0,

$$\int_{-4}^{8} f(x)dx = 2\left[\int_{-4}^{x_0} f(x)dx - \frac{1}{2}(x_0 - 1)f(x_0)\right] \tag{2}$$

Solving, we get,

$$\frac{x_0^4}{2} - \frac{32x_0^3}{3} + 29x_0^2 - 868x_0 + 64 \times \frac{88}{3} = 0 \tag{3}$$

$x_0 = 2$ yeilds slightly positive value and $x_0 = 3$ yeilds large negative value. Thus, the root the the equation lies very close to 2 but is higher than 2.

Note: The approximate solution turns out to be $x_0 = 2.2071$.

Marking Principle: By either way, student should state (with proper justification) that the x-coordinate is close to 2 but higher than 2.

γ. (a) (6 marks) Assume that human civilisation has carved out all the mantle and the central core of the earth, and started living on the inside surface of the earth's crust which is only about 7 km in thickness. As a consequence, people are finding that their weight has altered drastically. To feel normal weight again, one scientist suggested an idea involving tampering one of the motions of the earth. Can you describe this idea qualitatively and quantitatively?

Solution: For a person standing on the inside surface of the hollow shell, there is no effect of gravity due to the shell itself (shell theorem).
He / she would "feel" the weight purely due to the centrifugal force due to the rotation of earth. **(2 marks)**
Thus, to get the normal weight back, the earth must start rotating faster. **(0.5 mark)**
Shell thickness should be considered as negligible as compared to the radius of the earth. **(0.5 mark)**

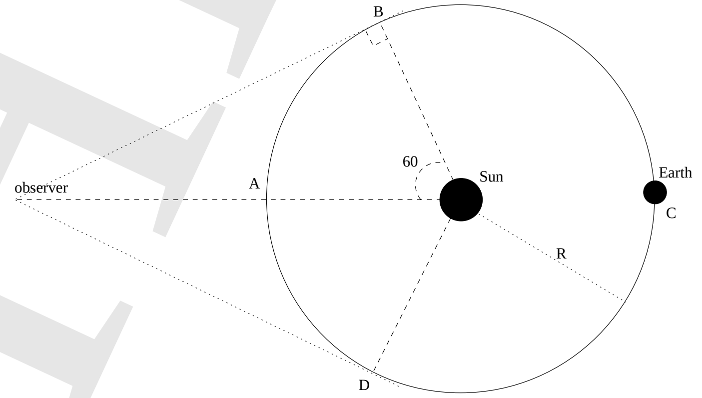
$$\begin{aligned}
g &= R\omega^2 \\
\therefore \omega &= \frac{2\pi}{T} = \sqrt{\frac{g}{R}} \\
\therefore T &= 2\pi\sqrt{\frac{R}{g}} \\
&\approx 2\pi\sqrt{\frac{6.4\times10^6}{10}} \\
&\approx 2\pi\times 800 \\
T &\approx 5000 \quad \text{sec}
\end{aligned}$$

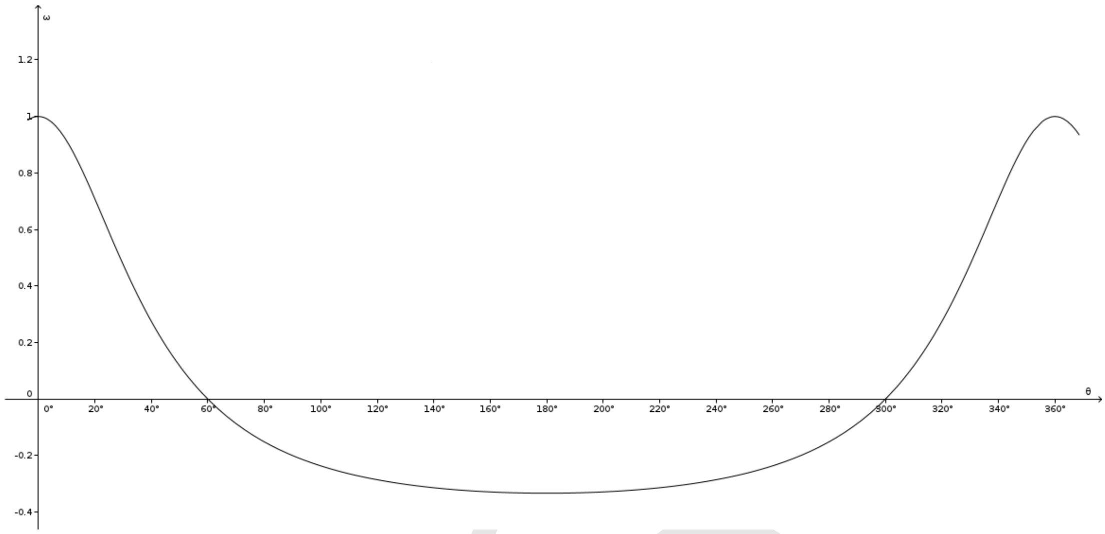
Thus, the earth's rotation period should be decreased to about 84 minutes. **(2 marks)**

This calculation is valid only for the equator and apparent weight at other latitudes would be different. **(1 mark)**

(b) (8 marks) An observer is sitting in a stationary spacecraft outside earth's orbit such that at the closest point, distance between the observer and the earth is 1 AU. Sketch a rough plot (exact functional form / shape not expected) showing how the orbital angular velocity of the earth varies with time (in an year), as seen by this observer. Mark every point of maximum or minimum clearly and write its coordinates. Similarly, mark every point with zero angular velocity and write its coordinates. Assume the orbit of earth around the Sun to be circular.

Solution:





The exact expression for the function above is

$$\omega(\theta) = \omega_0 \frac{\sin(\phi)\cos(\phi+\theta)}{\sin(\theta)}$$

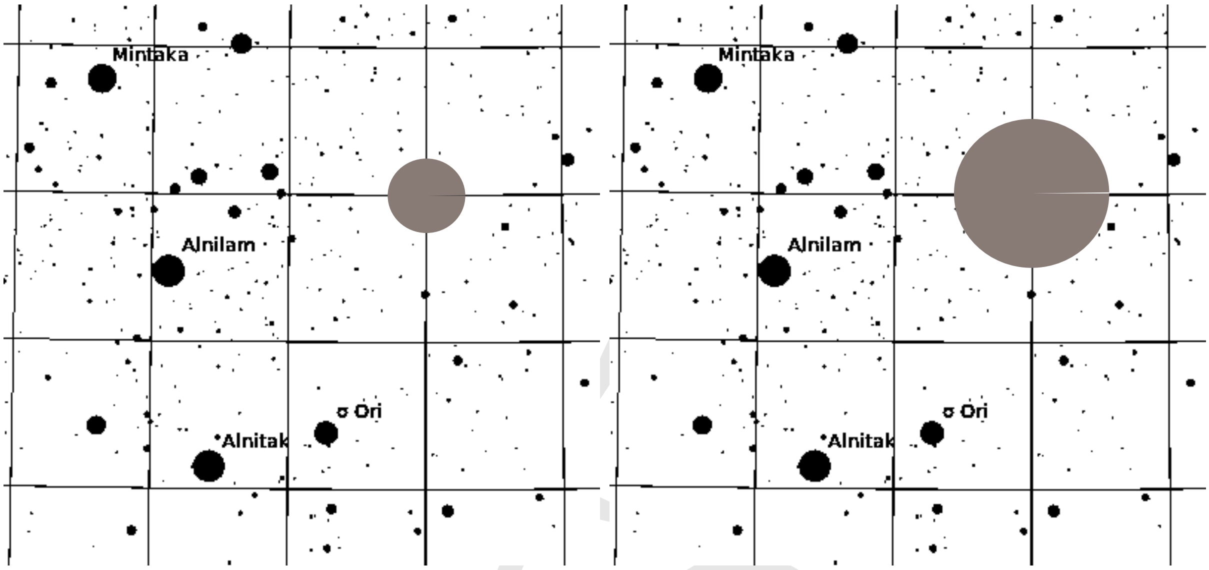
$$\text{where} \quad \tan\phi = \frac{R\sin(\theta)}{k + R(1-\cos(\theta))}$$

In this, R is the orbital radius of the earth (1 AU), k is the shortest separation between the observer and the earth orbit (also 1 AU), θ is the observer - sun - earth angle, ϕ is the earth - observer - sun angle. The coordinates for points of interest would be (0, ω_0), (60,0), (180, $\omega_0/3$) and (300,0), where x-axis represents θ. This θ can be recalibrated in terms of time with 30° interval corresponding to one month period.

The sign of ω as well as start of the year point are matters of convention. Hence if this graph is flipped on x-axis or is shifted along the x-axis, the answer is equally valid.

Marking scheme: 1.5 mark for each the four coordinates. 2 marks for the overall shape of the curve and proper markings of axes etc.

δ. In the 1957 science fiction novel "The black cloud" by Sir Fred Hoyle, an interstellar dark cloud is discovered appraching the solar system. We will try to retrace the steps explained in the novel to calculate speed and direction of the cloud. Two photographs given here show a small region inside the constellation of Orion. The photographs are taken exactly 2 months apart. The grid in the image has the size of $1^\circ \times 1^\circ$. By spectroscopy, it was realised that the emission line of neutral hydrogen (Rest wavelength 21.1000 cm) emitted by this cloud was showing up at 21.0789 cm wavelength.



(a) (4 marks) In how many months (after the second image) the cloud will entirely cover the belt of Orion (seen on the left)?

(b) (1 mark) Is the cloud headed directly for the solar system? Why?

(c) (2 marks) If yes, then in how many months (after the second image) the cloud will arrive at the earth? If no, what will be the closest separation between the cloud and the earth? Assume the cloud to have uniform velocity throughout the journey.

(d) (5 marks) If this spherically symmetric cloud is placed at the exact centre of the solar system, which planets will be engulfed by the cloud?

Note: The expression of relativistic Doppler effect is given by $\lambda_{\text{observed}} = \lambda_{\text{emitted}} \sqrt{\frac{c-v}{c+v}}$

Solution: (a) The size of the cloud is about 1 cm in the first image and 2 cm in the second image. Now, Mintaka and Alnitak are almost equidistant from the cloud's centre (about 4.6 cm) so they would be get covered at almost same time. If the physical diameter of cloud is x and its distace from us is d,

$$\begin{aligned} x &= \theta d = 2\theta(d - vt_2) = 9.2\theta(d - vt_3) \\ \therefore d &= 2vt_2 \\ \& \ 9.2vt_3 &= 7.2d + 2vt_2 = 16.4vt_2 \\ t_3 &= \frac{16.4}{9.2} \times 2 \ \text{ months} \\ &= 3.565 \ \text{ months} \end{aligned}$$

i.e. In just over six weeks from the second image (about 47 days), the belt of Orion will be covered by the cloud.

(b) The cloud is headed directly for the solar system as the position of its centre has not shifted from the first image to the second image.

(c)

$$\begin{aligned} d &= 2vt_2 = vt_4 \\ \therefore t_4 &= 2t_2 \\ t_4 &= 4 \text{ months} \end{aligned}$$

Thus, the cloud will arrive at the solar system in exactly 2 months after the second image is taken.
(d) Velocity of the cloud is given by,

$$\begin{aligned} \frac{c-v}{c+v} &= \left(\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}}\right)^2 = \left(\frac{21.1 \times (1 - 0.001}{21.1}\right)^2 = 0.998 \\ \therefore v &= \frac{2c}{1998} \approx 300 km/s \end{aligned}$$

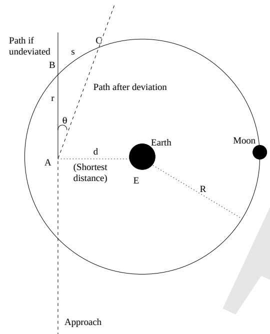
As per the grid, about 1.95 cm correspond to 1° separation in the sky. Thus, $\theta \approx 0.5^\circ$ in the first image.

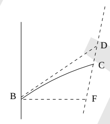
$$\begin{aligned} x &= d\theta = 2vt_2\theta \\ \therefore x &= \frac{2 \times 3 \times 10^5 \times 2 \times 30 \times 86400}{1.5 \times 10^{11}} \times 0.5 \times \frac{\pi}{180} \\ &\approx \frac{24 \times 8.64\pi}{360} \\ &\approx 1.728 \times \frac{\pi}{3} \\ x &\approx 1.8 \text{ A.U.} \end{aligned}$$

Thus, the cloud radius would be about 0.9 A. U. Meaning, the cloud will engulf Mercury and Venus.

ϵ. (12 marks) Let us assume a situation where an asteroid passes close to the earth. Some times the shortest distance of such an asteroid can be even lesser than the earth-moon distance. In such cases, the asteroid will cross the lunar orbit at two points. A particular asteroid makes a close-earth pass such that at its closest proximity to the earth, its distance from the earth is 256000 km. For simplicity, let us assume that the gravitational force of the earth acts on this asteriod only for a short duration of 1000 seconds, when the asteroid is exactly at the closest proximity point. Let us assume that the asteroid follows straight line paths with uniform velocities before and after this gravitational interaction and the velocity of the asteroid before the interaction was 6.25 km/s. Find out how much would be the deviation measured (in km) along the lunar orbit. Assume the Moon to be on the other side of the Earth in its orbit during the asteroid's transit, hence not having any impact on the motion. Take the orbit of the Moon around the Earth to be circular.

Solution:





(3 marks)

$$
\begin{aligned}
\vec{v} &= \vec{u} + \vec{a}t \\
\vec{v} &= u\hat{j} + \frac{GMt}{d^2}\hat{i} \\
&= 6.25 \times 10^3\hat{j} + \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^3}{(2.56 \times 10^8)^2}\hat{i} \\
&\approx 6250\hat{j} + 6.1\hat{i} \qquad (\textbf{3marks})
\end{aligned}
$$

As the first term on the R.H.S. is much larger as compared to the second term, the angle of deviation of the asteroid would be very small. In the figure, E denotes centre of the Earth, A closest proximity position of asteroid, BC is arc measured along the lunar orbit. AB is denoted by r and both EB and EC would equal to R. As angle of deviation is very small, Arc length BC can be approximated as length BD. Let angle BAC be θ and angle AEB be ϕ. Thus, angle FBC and hence approximately angle FBD will also be $90° - \phi$. Angle BFD can be approximated to $90°$.

$$
\begin{aligned}
\tan(\theta) &= \frac{at}{u} \approx 10^{-3} \\
\therefore \tan(\theta) &\approx \approx 10^{-3} \\
\sin(\phi) &= \frac{r}{R} \\
BD &= \frac{BF}{\cos(90° - \phi)} \\
&= \frac{r\tan(\theta)}{\sin(\phi)} \\
&= R\tan(\theta) \\
&\approx 3.84 \times 10^8 \times 10^{-3} \\
s &\approx 384km \qquad (\textbf{6marks})
\end{aligned}
$$

Note: If BC is approximated to BF, only 3 marks would be awarded. Secondly, $AB = r \neq R$. That approach does not carry any points.

Space for Rough Work