

1. Let Γ be a circle with centre O . Let Λ be another circle passing through O and intersecting Γ at points A and B . A diameter CD of Γ intersects Λ at a point P different from O . Prove that

$$\angle APC = \angle BPD.$$

2. Determine the smallest prime that does not divide any five-digit number whose digits are in a strictly increasing order.
3. Given real numbers $a, b, c, d, e > 1$ prove that

$$\frac{a^2}{c-1} + \frac{b^2}{d-1} + \frac{c^2}{e-1} + \frac{d^2}{a-1} + \frac{e^2}{b-1} \geq 20.$$

4. Let x be a non-zero real number such that $x^4 + \frac{1}{x^4}$ and $x^5 + \frac{1}{x^5}$ are both rational numbers. Prove that $x + \frac{1}{x}$ is a rational number.
5. In a triangle ABC , let H denote its orthocentre. Let P be the reflection of A with respect to BC . The circumcircle of triangle ABP intersects the line BH again at Q , and the circumcircle of triangle ACP intersects the line CH again at R . Prove that H is the incentre of triangle PQR .
6. Suppose that the vertices of a regular polygon of 20 sides are coloured with three colours – red, blue and green – such that there are exactly three red vertices. Prove that there are three vertices A, B, C of the polygon having the same colour such that triangle ABC is isosceles.

————— ★ ★ ★ ★ —————