Regional Mathematical Olympiad-2015

Time: 3 hours **December 06, 2015**

Instructions:

- **Calculators (in any form) and protractors are not allowed.**
- **Rulers and compasses are allowed.**
- **Answer all the questions.**
- **All questions carry equal marks. Maximum marks: 102.**
- **Answer to each question should start on a new page. Clearly indicate the question number.**

1. Two circles Γ and Σ, with centres O and O', respectively, are such that O' lies on Γ. Let A be a point on Σ and M the midpoint of the segment AO'. If B is a point on Σ different from A such that AB is parallel to OM, show that the midpoint of AB lies on Γ.

2. Let $P(x) = x^2 + ax + b$ be a quadratic polynomial where a and b are real numbers. Suppose $\langle P(-1)^2, P(0)^2, P(1)^2 \rangle$ is an arithmetic progression of integers. Prove that a and b are integers.

3. Show that there are infinitely many triples (x, y, z) of integers such that $x^3 + y^4 = z^{31}$.

4. Suppose 36 objects are placed along a circle at equal distances. In how many ways can 3 objects be chosen from among them so that no two of the three chosen objects are adjacent nor diametrically opposite?

5. Let ABC be a triangle with circumcircle Γ and incentre I. Let the internal angle bisectors of $\angle A$, $\angle B$ and $\angle C$ meet Γ in A', B' and C' respectively. Let $B'C'$ intersect AA' in P and AC in Q, and let BB' intersect AC in R. Suppose the quadrilateral $PIRQ$ is a kite; that is, $IP = IR$ and $QP = QR$. Prove that ABC is an equilateral triangle.

6. Show that there are infinitely many positive real numbers a which are not integers such that $a(a - 3\{a\})$ is an integer. (Here $\{a\}$ denotes the fractional part of a. For example $\{1.5\} = 0.5$; $\{-3.4\} = 0.6$.)

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